Preprint typeset in JHEP style - HYPER VERSION

Feynman rules for the rational part of the Electroweak 1-loop amplitudes

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Abstract: We present the complete set of Feynman rules producing the rational terms of kind R_2 needed to perform any 1-loop calculation in the Electroweak Standard Model. Our results are given both in the 't Hooft-Veltman and in the Four Dimensional Helicity regularization schemes. We also verified, by using both the 't Hooft-Feynman gauge and the Background Field Method, a huge set of Ward identities -up to 4-points- for the complete rational part of the Electroweak amplitudes. This provides a stringent check of our results and, as a by-product, an explicit test of the gauge invariance of the Four Dimensional Helicity regularization scheme in the complete Standard Model at 1-loop. The formulae presented in this paper provide the last missing piece for completely automatizing, in the framework of the OPP method, the 1-loop calculations in the $SU(3) \times SU(2) \times U(1)$ Standard Model.

Keywords: NLO, radiative corrections, LHC, ILC, Electroweak interactions.

Contents

1. Introduction

The complete automation of the 1-loop calculations for LHC and ILC physics is nowadays a feasible task [1]. The advent of the OPP reduction method [2], together with the concept of multiple cuts [3], allowed to revitalize the Unitarity Techniques [4], by reducing the computation of 1-loop amplitudes to a problem with the same conceptual complexity of a tree level calculation, resulting in achievements that were inconceivable only a few years ago [5].

The main idea of the OPP based techniques is directly extracting, from the 1-loop amplitude, the coefficients of the (known) scalar loop functions. This task can be reached in a completely numerical way by *opening the loop* and transforming the 1-loop amplitude in a tree level object with 2 more legs, that can be calculated, at the *integrand level*, by using the same recursion relations [6] that allow a very efficient computation of complicated multi-leg tree level processes [7, 8]. A second possible option is that one of the so-called Generalized Unitarity methods [9], where tree-level amplitudes are *glued together*.

Both procedures only allow the extraction of the Cut Constructible (CC) part of the amplitude in 4 dimensions, while a left over piece, the rational part R, needs to be derived separately. In the Generalized Unitarity approaches, that is achieved by computing the amplitude in different numbers of space-time dimensions, or via bootstrapping techniques [11], while, in the OPP approach, R is split in 2 pieces $\mathrm{R} = \mathrm{R}_1 + \mathrm{R}_2$. The first piece, R_1, is derivable in the same framework used to reconstruct the CC part of the amplitude, while R_2 is computable through a special set Feynman rules for the theory at hand [12], to be used in a tree level-like computation.

Such a set of R_2 Feynman rules has been already derived for QED in [12] and for QCD in [13], and it is the main aim of the present paper to present the complete set of the R_2 Feynman rules for the Standard Model (SM) of the Electroweak (EW) interactions. In addition, as a by-product, we use the derived formulae to explicitly check the gauge invariance of the Four Dimensional Helicity regularization scheme in the EW sector at 1-loop, the motivation being that this is a very well studied subject in QCD [14], but, in our knowledge, very little can be found in the literature for the full EW Standard Model.

The outline of the paper is as follows. In section 2 we remind some facts on the origin of R and on the splitting $\mathrm{R} = \mathrm{R}_1 + \mathrm{R}_2$. Section 3 contains the complete list of all possible special R_2 EW SM vertices in the 't Hooft-Feynman gauge and, in section 4, we describe the tests we performed on our formulae and our findings. Finally, our conclusions are drawn in section 5 and, in the appendix, we collect a list of Ward identities.

2. Theory, facts and conjectures on R, R_1 and R_2

Before carrying out our program, we spend a few words on the origin of R. Our starting point is the general expression for the *integrand* of a generic m-point one-loop (sub-) amplitude

$$\bar{A}(\bar{q}) = \frac{\bar{N}(\bar{q})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}\,, \quad \bar{D}_i = (\bar{q} + p_i)^2 - m_i^2\,, \tag{2.1}$$

where $\bar{q}$ is the integration momentum and where dimensional regularization is assumed, so that a bar denotes objects living in $n = 4 + \epsilon$ dimensions and a tilde represents ϵ-dimensional quantities. When a n-dimensional index is contracted with a 4-dimensional vector v_μ, the 4-dimensional part is automatically selected. For example

$$\bar{q} \cdot v \equiv (q + \tilde{q}) \cdot v = q \cdot v \,, \quad \bar{\not{v}} \equiv \bar{\gamma}_{\bar{\mu}} \, v^\mu = \not{v} \quad \text{and} \quad \bar{q}^2 = q^2 + \tilde{q}^2 \,. \tag{2.2}$$

The numerator function $\bar{N}(\bar{q})$ can be split into a 4-dimensional plus an ϵ-dimensional part

$$\bar{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}^2, q, \epsilon) \,. \tag{2.3}$$

$N(q)$ lives in 4 dimensions, and can be therefore expanded in terms of 4-dimensional denominators

$$D_i = (q + p_i)^2 - m_i^2 = \bar{D}_i - \tilde{q}^2 \,. \tag{2.4}$$

Some among the coefficients in this expansion are interpreted, in the OPP method, as the desired coefficients of the 1-loop scalar integrals and can be determined numerically, while the mismatch between this expansion in terms of 4-dimensional denominators, and the n-dimensional denominators appearing in eq. 2.1, is the origin of the rational terms R_1. There exist at least two ways [15, 16] to compute R_1, which allow to determine it by means of a purely numerical knowledge of the 4-dimensional CC part of the amplitude, while this does not seem to be possible for R_2, whose origin is the term $\tilde{N}(\tilde{q}^2, q, \epsilon)$ in eq. 2.3, after integration over the loop momentum:

$$\mathrm{R}_2 \equiv \frac{1}{(2\pi)^4} \int d^n \, \bar{q} \frac{\tilde{N}(\tilde{q}^2, q, \epsilon)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \,. \tag{2.5}$$

However, R_2 can be computed by extracting $\tilde{N}(\tilde{q}^2, q, \epsilon)$ from any given *integrand* $\bar{A}(\bar{q})$, which can be achieved by splitting, in the analytic expression of the numerator function, the n-dimensional integration momentum $\bar{q}$, the n-dimensional gamma matrices $\bar{\gamma}_{\bar{\mu}}$ and the n-dimensional metric tensor $\bar{g}^{\bar{\mu}\bar{\nu}}$ into a 4-dimensional component plus remaining pieces:

$$\begin{aligned} \bar{q} &= q + \tilde{q} \,, \\ \bar{\gamma}_{\bar{\mu}} &= \gamma_\mu + \tilde{\gamma}_{\tilde{\mu}} \,, \\ \bar{g}^{\bar{\mu}\bar{\nu}} &= g^{\mu\nu} + \tilde{g}^{\tilde{\mu}\tilde{\nu}} \,. \end{aligned} \tag{2.6}$$

Therefore, a practical way to determine R_2 is computing analytically, by means of Feynman diagrams, once for all and with the help of eq. 2.6, tree-level like Feynman rules, namely effective vertices, by calculating the R_2 part coming from all possible one-particle irreducible Green functions of the theory at hand, up to four external legs. The fact that four external legs are enough to account for R_2 is guaranteed by the ultraviolet nature of the rational terms, proved in [17]. This property does not hold, instead, for R_1, that, diagram by diagram, can give non vanishing contributions to any one-particle irreducible m-point function, because, even when finite, the tensor integrals generating R_1 are eventually expressed, via tensor reduction, in terms of linear combinations of 1-loop scalar functions

that can be ultraviolet divergent. This fact prevents the possibility of calculating a finite set of effective vertices reproducing R_1.

Eq. 2.5 generates a set of simple basic integrals with up to 4 denominators, containing powers of $\tilde{q}$ and ϵ in the numerator. A list that exhausts all possibilities in the $\xi = 1$ 't Hooft-Feynman gauge can be found in [13]. Notice, however, that, according to the chosen regularization scheme, results may differ. In eq. 2.5 we assume the 't Hooft-Veltman (HV) scheme, while in the Four Dimensional Helicity scheme (FDH), any explicit ϵ dependence in the numerator function is discarded before integration, such that

$$\mathrm{R}_2\Big|_{FDH} = \frac{1}{(2\pi)^4}\int d^n\,\bar{q}\frac{\tilde{N}(\tilde{q}^2,q,\epsilon=0)}{\bar{D}_0\bar{D}_1\cdots\bar{D}_{m-1}}\,. \tag{2.7}$$

The asymmetric role played by R_1 and R_2 is somewhat annoying. As we have seen, R_1 is directly connected with the (4-dimensional) CC part of the amplitude, and can be computed, even numerically, without any reference to Feynman diagrams, while R_2 requires an analytic determination in terms of Feynman diagrams, so that one would like to be able to put both pieces on the same footing. Unfortunately, no easy direct connection between R_2 and the CC part of the amplitude has been found so far (at least within our treatment at the *integrand level*) and, in the rest of this paragraph, we speculate a bit on this subject.

Reconstructing R_2 numerically would require to detect "signs" of it in the CC part. For example, one could naively think that, by looking at any q^2 in the CC part, the $\tilde{q}^2$ dependence could be inferred via the replacement

$$q^2 \to q^2 + \tilde{q}^2\,. \tag{2.8}$$

However, such a dependence is impossible to reconstruct numerically, when remaining in 4 dimensions, as it can be illustrated by considering the following simple 3-point sub-amplitude:

$$\mathrm{A} \equiv \frac{1}{(2\pi)^4}\int d^n\,\bar{q}\frac{(q\cdot\ell_3)(q\cdot\ell_4)}{\bar{D}_0\bar{D}_1\bar{D}_2}\,, \tag{2.9}$$

where

$$\ell_3^\mu = < \ell_1|\gamma^\mu|\ell_2]\,,\quad \ell_4^\mu = < \ell_2|\gamma^\mu|\ell_1] \quad \text{with} \quad \ell_{1,2}^2 = 0\,. \tag{2.10}$$

From the one hand, the 4-dimensional numerator $(q\cdot\ell_3)(q\cdot\ell_4)$ in eq. 2.9 does not contain any q^2 to be continued through the replacement of eq. 2.8. On the other hand, it can be manipulated as follows

$$(q\cdot\ell_3)(q\cdot\ell_4) = 4\,(q\cdot\ell_1)(q\cdot\ell_2) - 2\,q^2\,(\ell_1\cdot\ell_2)\,, \tag{2.11}$$

and now the shift of eq. 2.8 would produce a $\tilde{q}^2$ contribution, in disagreement with our previous finding. We therefore conclude that not enough information is present in the 4-dimensional part to reconstruct R_2. This is the reason why one is forced to work analytically in n dimensions to reconstruct the R_2 contribution [1].

[1] In other approaches [9], a numerical determination of the whole R contribution can be achieved, but at the price of explicitly computing numerically the amplitude in 4, 6 and 8 dimensions.

Nevertheless, based on a simple reasoning, one argues that some gauge invariance properties of the 4-dimensional part of the amplitude should be transferred to R_2. In fact, for physical processes, the sum of $\mathrm{R}_1 + \mathrm{R}_2$ is gauge invariant. On the other hand, R_1 can be fully reconstructed from the 4-dimensional, gauge invariant, CC part of the amplitude, meaning that, by changing gauge, the same expressions for R_1 should be found, and, as a consequence, also *the same result for* R_2. This should be off course only true for amplitudes with physical external particles, because different gauges may have, in general, a different content in terms unphysical external fields. Therefore one can conjecture that

The R_2 *part of a physical amplitude gives the same result when computed in any gauge* [2] .

This conjecture, being rather strong, should be proved with an actual calculation. Unfortunately, such a calculation would require to extend the set of basic integrals in [13] to be able to deal with non-renormalizable gauges. That is beyond the scope of this work, and we leave it for a future publication.

In the present paper, we fix the gauge to be the the 't Hooft-Feynman one and we derive all of the effective Electroweak R_2 Feynman rules by applying the splittings of eq. 2.6 Feynman diagram by Feynman diagram. For the interested reader, explicit examples of this technique can be found in [13].

3. Results

In this section, we give the complete list of the effective Electroweak vertices contributing to R_2 in the 't Hooft-Feynman gauge [3]. A parameter λ_{HV} is introduced in our formulae such that $\lambda_{HV} = 1$ corresponds to the 't Hooft-Veltman scheme and $\lambda_{HV} = 0$ to the FDH scheme of eq. 2.7. We used the Feynman rules given in [19] and our notations are as follows: $l_1 = e$, $l_2 = \mu$, $l_3 = \tau$, $l_4 = \nu_e$, $l_5 = \nu_\mu$, $l_6 = \nu_\tau$ and $q_1 = d$, $q_2 = u$, $q_3 = s$, $q_4 = c$, $q_5 = b$, $q_6 = t$. In addition, $e_1 = e$, $e_2 = \mu$, $e_3 = \tau$, $\nu_1 = \nu_e$, $\nu_2 = \nu_\mu$, $\nu_3 = \nu_\tau$ and $u_1 = u$, $u_2 = c$, $u_3 = t$, $d_1 = d$, $d_2 = s$, $d_3 = b$. When appearing as external particles, l, ν_l, u and d stand for the three charged leptons, the three (massless) neutrinos, the three up-type quarks and the three down-type quarks, respectively. Effective vertices with external quarks are always understood to be diagonal in the color space. Finally, N_{col} is the number of colors and $V_{u_i d_j}$ are CKM matrix elements. Occasionally, combinations such as

$$\sum_{i,j=1}^{3} \left(V_{u_i d_j} V^\dagger_{d_j u_i} \right) = 3 \quad \text{and} \quad \sum_{i=1}^{3} 1 = 3$$

appear in our formulae. In such cases, we do not explicitly work out the sum in order to make our results also readable family by family.

A last comment is in order with respect to our treatment of γ_5 in vertices containing fermionic lines. When computing all contributing Feynman diagrams, we pick up a

[2]This does not mean that the R_2 part of the Green functions satisfy the Ward identities separately from R_1, as we have checked explicitly.

[3]They can be also found as a `FORM` [18] output in http://www.ugr.es/local/pittau/CutTools.

"special" vertex in the loop and anticommute all γ_5's to reach it before performing the n-dimensional algebra, and, when a trace is present, we start reading it from this vertex. This treatment produces, in general, a term proportional to the totally antisymmetric ϵ tensor, whose coefficient may be different depending on the choice of the "special" vertex. However, when summing over all quantum numbers of each fermionic family, we checked that all contributions proportional to ϵ cancel. In addition, we explicitly verified that our results satisfy the large set of Ward identities given in appendix A.

3.1 Electroweak effective vertices with 2 external legs

In this section, we give the complete list of the non vanishing 2-point R_2 effective vertices.

3.1.1 Scalar-Scalar effective vertices

The generic effective vertex is

$$\boxed{S_1 \text{ - - - - - } \bullet \text{ - - - - - } S_2 \;=\; \frac{ie^2}{16\pi^2 s_w^2} C}$$

with the actual values of S_1, S_2 and C

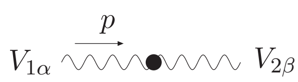
$$
\begin{aligned}
H\chi \quad &: \quad C = 0 \\
HH \quad &: \quad C = \frac{m_\phi^2}{4} + \frac{m_\chi^2}{8c_w^2} + \frac{1 - 12\lambda_{HV}}{4}\left(1 + \frac{1}{2c_w^4}\right) m_W^2 - \left(1 + \frac{1}{2c_w^2}\right)\frac{p^2}{12} + K \\
\chi\chi \quad &: \quad C = \frac{m_\phi^2}{4} + \frac{m_H^2}{8c_w^2} + \frac{1 - 4\lambda_{HV}}{4}\left(1 + \frac{1}{2c_w^4}\right) m_W^2 - \left(1 + \frac{1}{2c_w^2}\right)\frac{p^2}{12} + K \\
\phi^-\phi^+ \quad &: \quad C = \frac{m_H^2 + m_\chi^2}{8} + \frac{(3 - 4\lambda_{HV})\, c_w^4 - 2c_w^2 + \left(\frac{1}{2} - 2\lambda_{HV}\right)}{c_w^4}\frac{m_W^2}{4} + \frac{m_\phi^2}{8c_w^2} \\
&\quad - \left(1 + \frac{1}{2c_w^2}\right)\frac{p^2}{12} + \frac{1}{2m_W^2}\left[\sum_{i=1}^{3}\left(m_{e_i}^2\left(m_{e_i}^2 - \frac{p^2}{3}\right)\right)\right. \\
&\quad \left. + \; N_{\mathrm{col}}\sum_{i,j=1}^{3}\left(V_{u_i d_j} V^\dagger_{d_j u_i}\left(m_{u_i}^2 + m_{d_j}^2\right)\left(m_{u_i}^2 + m_{d_j}^2 - \frac{p^2}{3}\right)\right)\right]
\end{aligned}
\tag{3.1}
$$

where

$$
K = \frac{1}{m_W^2}\left[\sum_{i=1}^{6}\left(m_{l_i}^2\left(m_{l_i}^2 - \frac{p^2}{6}\right)\right) + N_{\mathrm{col}}\sum_{i=1}^{6}\left(m_{q_i}^2\left(m_{q_i}^2 - \frac{p^2}{6}\right)\right)\right] \tag{3.2}
$$

3.1.2 Vector-Vector effective vertices

The generic effective vertex is



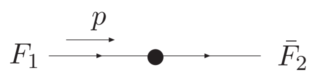
$$= \frac{ie^2}{\pi^2}\left(C_1 p_\alpha p_\beta + C_2 g_{\alpha\beta}\right)$$

with the actual values of V_1, V_2, C_1 and C_2

$$\begin{aligned}
AA \quad &: \quad C_1 = -\frac{1}{24}\lambda_{HV} \\
&\quad\;\; C_2 = \frac{1}{8}\left[p^2\left(\frac{1}{6}+\frac{\lambda_{HV}}{3}\right) - m_W^2\right] - \frac{1}{4}\left[\sum_{i=1}^{6}\left(Q_{l_i}^2\left(m_{l_i}^2 - \frac{1}{6}p^2\right)\right)\right. \\
&\qquad\qquad \left. + \; N_{\rm col}\sum_{i=1}^{6}\left(Q_{q_i}^2\left(m_{q_i}^2 - \frac{1}{6}p^2\right)\right)\right] \\
AZ \quad &: \quad C_1 = \frac{1}{24}\frac{c_w}{s_w}\lambda_{HV} \\
&\quad\;\; C_2 = -\frac{1}{8}\frac{c_w}{s_w}\left[p^2\left(\frac{1}{6}+\frac{\lambda_{HV}}{3}\right) - m_W^2\right] + \frac{1}{4c_w}\left[\sum_{i=1}^{6}\left(\left(\frac{Q_{l_i}I_{3l_i}}{2s_w} - Q_{l_i}^2 s_w\right)\right.\right. \\
&\qquad\qquad \times \; \left.\left(m_{l_i}^2 - \frac{1}{6}p^2\right)\right) + N_{\rm col}\sum_{i=1}^{6}\left.\left(\left(\frac{Q_{q_i}I_{3q_i}}{2s_w} - Q_{q_i}^2 s_w\right)\left(m_{q_i}^2 - \frac{1}{6}p^2\right)\right)\right] \\
ZZ \quad &: \quad C_1 = -\frac{1}{24}\frac{c_w^2}{s_w^2}\lambda_{HV} \\
&\quad\;\; C_2 = \frac{1}{8}\frac{c_w^2}{s_w^2}\left[p^2\left(\frac{1}{6}+\frac{\lambda_{HV}}{3}\right) - m_W^2\right] + \frac{1}{4c_w^2}\left[\sum_{i=1}^{6}\left(\left(Q_{l_i}I_{3l_i} - \frac{I_{3l_i}^2}{2s_w^2} - Q_{l_i}^2 s_w^2\right)\right.\right. \\
&\qquad\qquad \times \; \left.\left(m_{l_i}^2 - \frac{1}{6}p^2\right)\right) + N_{\rm col}\sum_{i=1}^{6}\left.\left(\left(Q_{q_i}I_{3q_i} - \frac{I_{3q_i}^2}{2s_w^2} - Q_{q_i}^2 s_w^2\right)\left(m_{q_i}^2 - \frac{1}{6}p^2\right)\right)\right] \\
W^-W^+ \quad &: \quad C_1 = -\frac{1}{24 s_w^2}\lambda_{HV} \\
&\quad\;\; C_2 = \frac{1}{8s_w^2}\left[p^2\left(\frac{1}{6}+\frac{\lambda_{HV}}{3}\right) - m_W^2\right] - \frac{1}{32s_w^2}\left[\sum_{i=1}^{3}\left(m_{e_i}^2 - \frac{p^2}{3}\right)\right. \\
&\qquad\qquad \left. + \; N_{\rm col}\sum_{i,j=1}^{3}\left(V_{u_i d_j}V_{d_j u_i}^\dagger\left(m_{u_i}^2 + m_{d_j}^2 - \frac{p^2}{3}\right)\right)\right]
\end{aligned} \tag{3.3}$$

3.1.3 Fermion-Fermion effective vertices

The generic effective vertex is



$$= \frac{ie^2}{\pi^2}\left[\left(C_-\Omega^- + C_+\Omega^+\right)\not{p} + C_0\right]\lambda_{HV}$$

with the actual values of F_1, $\bar{F}_2$, C_-, C_+ and C_0

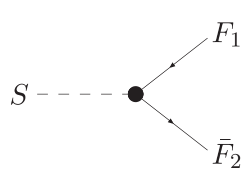
$$
\begin{aligned}
u\bar{u} \quad : \quad C_- &= \frac{1}{16}\frac{Q_u^2}{c_w^2} \\
C_+ &= \frac{1}{16}\left(\frac{I_{3u}^2}{s_w^2 c_w^2} - \frac{2Q_u I_{3u}}{c_w^2} + \frac{Q_u^2}{c_w^2} + \frac{1}{2s_w^2}\sum_{j=1}^{3}\left(V_{ud_j}V_{d_j u}^\dagger\right)\right) \\
C_0 &= \frac{m_u Q_u}{8c_w^2}\left(Q_u - I_{3u}\right) \\
d\bar{d} \quad : \quad C_- &= \frac{1}{16}\frac{Q_d^2}{c_w^2} \\
C_+ &= \frac{1}{16}\left(\frac{I_{3d}^2}{s_w^2 c_w^2} - \frac{2Q_d I_{3d}}{c_w^2} + \frac{Q_d^2}{c_w^2} + \frac{1}{2s_w^2}\sum_{i=1}^{3}\left(V_{u_i d}V_{d u_i}^\dagger\right)\right) \\
C_0 &= \frac{m_d Q_d}{8c_w^2}\left(Q_d - I_{3d}\right) \\
l\bar{l} \quad : \quad C_- &= \frac{1}{16}\frac{Q_l^2}{c_w^2} \\
C_+ &= \frac{1}{16}\left(\frac{I_{3l}^2}{s_w^2 c_w^2} - \frac{2Q_l I_{3l}}{c_w^2} + \frac{Q_l^2}{c_w^2} + \frac{1}{2s_w^2}\right) \\
C_0 &= \frac{m_l Q_l}{8c_w^2}\left(Q_l - I_{3l}\right) \\
\nu_l\bar{\nu}_l \quad : \quad C_- &= 0 \\
C_+ &= \frac{1}{32s_w^2}\left(\frac{1}{2c_w^2} + 1\right) \\
C_0 &= 0
\end{aligned}
\tag{3.4}
$$

3.2 Electroweak effective vertices with 3 external legs

We list here the 3-point R_2 effective vertices.

3.2.1 Scalar-Fermion-Fermion effective vertices

The generic effective vertex is



$$= \frac{e^3}{\pi^2}(C_-\Omega^- + C_+\Omega^+)$$

with the actual values of S, F_1, $\bar{F}_2$, C_- and C_+

$$
\begin{aligned}
Hu\bar{u} \quad : \quad C_- &= \frac{im_u}{8m_W s_w}\left[\frac{(1+\lambda_{HV})\,Q_u^2}{2c_w^2} + \frac{1}{16s_w^2}\sum_{j=1}^{3}\left(V_{ud_j}V^\dagger_{d_j u}\right) + \frac{I_{3u}}{c_w^2}\left(\frac{I_{3u}}{8s_w^2}\right.\right. \\
&\quad \left.\left. - \frac{(1+\lambda_{HV})\,Q_u}{2}\right) + \frac{1}{16m_W^2 s_w^2}\sum_{j=1}^{3}\left(m_{d_j}^2 V_{ud_j}V^\dagger_{d_j u}\right)\right] \\
C_+ &= C_-
\end{aligned}
$$

$$
\begin{aligned}
Hd\bar{d} \quad : \quad C_- &= \frac{im_d}{8m_W s_w}\left[\frac{(1+\lambda_{HV})\,Q_d^2}{2c_w^2} + \frac{1}{16s_w^2}\sum_{i=1}^{3}\left(V_{u_i d}V^\dagger_{du_i}\right) + \frac{I_{3d}}{c_w^2}\left(\frac{I_{3d}}{8s_w^2}\right.\right. \\
&\quad \left.\left. - \frac{(1+\lambda_{HV})\,Q_d}{2}\right) + \frac{1}{16m_W^2 s_w^2}\sum_{i=1}^{3}\left(m_{u_i}^2 V_{u_i d}V^\dagger_{du_i}\right)\right] \\
C_+ &= C_-
\end{aligned}
$$

$$
\begin{aligned}
Hl\bar{l} \quad : \quad C_- &= \frac{im_l}{8m_W s_w}\left[\frac{(1+\lambda_{HV})\,Q_l^2}{2c_w^2} + \frac{1}{16s_w^2} + \frac{I_{3l}}{c_w^2}\left(\frac{I_{3l}}{8s_w^2} - \frac{(1+\lambda_{HV})\,Q_l}{2}\right)\right] \\
C_+ &= C_-
\end{aligned}
$$

$$
\begin{aligned}
H\nu_l\bar{\nu}_l \quad : \quad C_- &= 0 \\
C_+ &= 0
\end{aligned}
$$

$$
\begin{aligned}
\chi u\bar{u} \quad : \quad C_- &= -\frac{m_u}{4m_W s_w}\left[\frac{(1+\lambda_{HV})\,Q_u^2 I_{3u}}{2c_w^2} + \frac{1}{32s_w^2}\sum_{j=1}^{3}\left(V_{ud_j}V^\dagger_{d_j u}\right) + \frac{I_{3u}}{c_w^2}\left(\frac{1}{32s_w^2}\right.\right. \\
&\quad \left.\left. - \frac{(1+\lambda_{HV})\,Q_u I_{3u}}{2}\right) - \frac{1}{16m_W^2 s_w^2}\sum_{j=1}^{3}\left(m_{d_j}^2 I_{3d_j} V_{ud_j}V^\dagger_{d_j u}\right)\right] \\
C_+ &= -C_-
\end{aligned}
$$

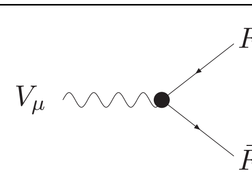
$$
\begin{aligned}
\chi d\bar{d} \quad : \quad C_- &= -\frac{m_d}{4m_W s_w}\left[\frac{(1+\lambda_{HV})\,Q_d^2 I_{3d}}{2c_w^2} - \frac{1}{32s_w^2}\sum_{i=1}^{3}\left(V_{u_i d}V^\dagger_{du_i}\right) + \frac{I_{3d}}{c_w^2}\left(\frac{1}{32s_w^2}\right.\right. \\
&\quad \left.\left. - \frac{(1+\lambda_{HV})\,Q_d I_{3d}}{2}\right) - \frac{1}{16m_W^2 s_w^2}\sum_{i=1}^{3}\left(m_{u_i}^2 I_{3u_i} V_{u_i d}V^\dagger_{du_i}\right)\right] \\
C_+ &= -C_-
\end{aligned}
$$

$$
\begin{aligned}
\chi l\bar{l} \quad : \quad C_- &= -\frac{m_l}{4m_W s_w}\left[\frac{(1+\lambda_{HV})\,Q_l^2 I_{3l}}{2c_w^2} - \frac{1}{32s_w^2} + \frac{I_{3l}}{c_w^2}\left(\frac{1}{32s_w^2} - \frac{(1+\lambda_{HV})\,Q_l I_{3l}}{2}\right)\right. \\
&\quad \left. - \frac{m_l^2 I_{3l}}{8m_W^2 s_w^2}\left(-\frac{1}{4} + I_{3l}^2\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& & C_+ &= -C_- \\[1em]
\chi\nu_l\bar{\nu}_l &: & C_- &= 0 \\
& & C_+ &= 0 \\[1em]
\phi^- u\bar{d} &: & C_- &= -\frac{i m_d V^\dagger_{du}}{4\sqrt{2} m_W s_w}\left[\frac{1}{c_w^2}\left(\frac{-1}{16} - \frac{(1+\lambda_{HV})\, Q_u Q_d}{2}\right) - \frac{3}{32 s_w^2}\right. \\
& & &\left. -\ \frac{m_u^2}{16 s_w^2 m_W^2} + \frac{I_{3u}}{c_w^2}\left(\frac{(1+\lambda_{HV})\, Q_d}{2} + \frac{1}{16}\right)\right] \\
& & C_+ &= \frac{i m_u V^\dagger_{du}}{4\sqrt{2} m_W s_w}\left[\frac{1}{c_w^2}\left(\frac{-1}{16} - \frac{(1+\lambda_{HV})\, Q_u Q_d}{2}\right) - \frac{3}{32 s_w^2}\right. \\
& & &\left. -\ \frac{m_d^2}{16 s_w^2 m_W^2} + \frac{I_{3d}}{c_w^2}\left(\frac{(1+\lambda_{HV})\, Q_u}{2} - \frac{1}{16}\right)\right] \\[1em]
\phi^+ d\bar{u} &: & C_- &= -\frac{i m_u V_{ud}}{4\sqrt{2} s_w m_W}\left[\frac{1}{c_w^2}\left(\frac{1}{16} + \frac{(1+\lambda_{HV})\, Q_u Q_d}{2}\right) + \frac{3}{32 s_w^2}\right. \\
& & &\left. +\ \frac{m_d^2}{16 s_w^2 m_W^2} - \frac{I_{3d}}{c_w^2}\left(\frac{(1+\lambda_{HV})\, Q_u}{2} - \frac{1}{16}\right)\right] \\
& & C_+ &= \frac{i m_d V_{ud}}{4\sqrt{2} m_W s_w}\left[\frac{1}{c_w^2}\left(\frac{1}{16} + \frac{(1+\lambda_{HV})\, Q_u Q_d}{2}\right) + \frac{3}{32 s_w^2}\right. \\
& & &\left. +\ \frac{m_u^2}{16 s_w^2 m_W^2} - \frac{I_{3u}}{c_w^2}\left(\frac{(1+\lambda_{HV})\, Q_d}{2} + \frac{1}{16}\right)\right] \\[1em]
\phi^- \nu_l \bar{l} &: & C_- &= -\frac{i m_l}{4\sqrt{2} m_W s_w}\left[\frac{Q_l}{16 c_w^2} - \frac{3}{32 s_w^2} + \frac{I_{3\nu_l}}{c_w^2}\left(\frac{(1+\lambda_{HV})\, Q_l}{2} + \frac{1}{16}\right)\right] \\
& & C_+ &= 0 \\[1em]
\phi^+ l\bar{\nu}_l &: & C_- &= 0 \\
& & C_+ &= \frac{i m_l}{4\sqrt{2} m_W s_w}\left[-\frac{Q_l}{16 c_w^2} + \frac{3}{32 s_w^2} - \frac{I_{3\nu_l}}{c_w^2}\left(\frac{(1+\lambda_{HV})\, Q_l}{2} + \frac{1}{16}\right)\right] \qquad (3.5)
\end{aligned}
$$

3.2.2 Vector-Fermion-Fermion effective vertices

The generic effective vertex is



$$= \frac{ie^3}{\pi^2}(C_-\Omega^- + C_+\Omega^+)\gamma_\mu$$

with the actual values of V, F_1, $\bar{F}_2$, C_- and C_+

$$
\begin{aligned}
Au\bar{u} \quad : \quad C_- &= \frac{1}{4}\left[\frac{(1+\lambda_{HV})\,Q_u^3}{4c_w^2} + \frac{m_u^2}{8s_w^2 m_W^2}\left(\frac{1}{2}\sum_{j=1}^{3}\left(V_{ud_j}V^\dagger_{d_j u}Q_{d_j}\right)\right.\right. \\
&+ \left.\left.\frac{Q_u}{4} + Q_u I_{3u}^2\right)\right] \\
C_+ &= \frac{1}{4}\left[\frac{(1+\lambda_{HV})\,Q_u^3}{4c_w^2} - \frac{(1+\lambda_{HV})\,Q_u^2 I_{3u}}{2c_w^2} + \frac{(1+\lambda_{HV})\,Q_u I_{3u}^2}{4s_w^2 c_w^2}\right. \\
&+ \frac{1}{4s_w^2}\left(\frac{1}{4m_W^2}\sum_{j=1}^{3}\left(V_{ud_j}V^\dagger_{d_j u}m_{d_j}^2 Q_{d_j}\right)\right. \\
&\left.\left.+\frac{m_u^2 Q_u\left(1+4I_{3u}^2\right)}{8m_W^2} + \sum_{j=1}^{3}\left(V_{ud_j}V^\dagger_{d_j u}\left(1+Q_{d_j}\right)\frac{(1+\lambda_{HV})}{2}\right)\right)\right] \\
Ad\bar{d} \quad : \quad C_- &= \frac{1}{4}\left[\frac{(1+\lambda_{HV})\,Q_d^3}{4c_w^2} + \frac{m_d^2}{8s_w^2 m_W^2}\left(\frac{1}{2}\sum_{i=1}^{3}\left(V_{u_i d}V^\dagger_{d u_i}Q_{u_i}\right)\right.\right. \\
&+ \left.\left.\frac{Q_d}{4} + Q_d I_{3d}^2\right)\right] \\
C_+ &= \frac{1}{4}\left[\frac{(1+\lambda_{HV})\,Q_d^3}{4c_w^2} - \frac{(1+\lambda_{HV})\,Q_d^2 I_{3d}}{2c_w^2} + \frac{(1+\lambda_{HV})\,Q_d I_{3d}^2}{4s_w^2 c_w^2}\right. \\
&+ \frac{1}{4s_w^2}\left(\frac{1}{4m_W^2}\sum_{i=1}^{3}\left(V_{u_i d}V^\dagger_{d u_i}m_{u_i}^2 Q_{u_i}\right)\right. \\
&\left.\left.+\frac{m_d^2 Q_d\left(1+4I_{3d}^2\right)}{8m_W^2} + \sum_{i=1}^{3}\left(V_{u_i d}V^\dagger_{d u_i}\left(Q_{u_i}-1\right)\frac{(1+\lambda_{HV})}{2}\right)\right)\right] \\
Al\bar{l} \quad : \quad C_- &= \frac{1}{4}\left[\frac{(1+\lambda_{HV})\,Q_l^3}{4c_w^2} + \frac{m_l^2}{8s_w^2 m_W^2}\left(\frac{Q_l}{4} + Q_l I_{3l}^2\right)\right] \\
C_+ &= \frac{1}{4}\left[\frac{(1+\lambda_{HV})\,Q_l^3}{4c_w^2} - \frac{(1+\lambda_{HV})\,Q_l^2 I_{3l}}{2c_w^2} + \frac{(1+\lambda_{HV})\,Q_l I_{3l}^2}{4s_w^2 c_w^2}\right. \\
&\left.+ \frac{1}{4s_w^2}\left(\frac{m_l^2 Q_l\left(1+4I_{3l}^2\right)}{8m_W^2} - \frac{(1+\lambda_{HV})}{2}\right)\right] \\
A\nu_l\bar{\nu}_l \quad : \quad C_- &= 0 \\
C_+ &= \frac{1}{32s_w^2}\left[\frac{m_l^2 Q_l}{2m_W^2} + (Q_l+1)(1+\lambda_{HV})\right] \\
Zu\bar{u} \quad : \quad C_- &= \frac{1}{8c_w}\left\{\frac{(1+\lambda_{HV})\,Q_u^3 s_w}{2c_w^2} + \frac{m_u^2}{8s_w m_W^2}\left[\sum_{j=1}^{3}\left(V_{ud_j}V^\dagger_{d_j u}\left(Q_{d_j} - \frac{I_{3d_j}}{s_w^2}\right)\right)\right.\right. \\
&\left.\left.+\left(Q_u - \frac{I_{3u}}{s_w^2}\right)\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
C_+ &= \frac{1}{8c_w}\left\{\frac{(1+\lambda_{HV})\,Q_u^3 s_w}{2c_w^2} - \frac{(1+\lambda_{HV})\,Q_u^2 I_{3u}(1+2s_w^2)}{2s_w c_w^2}\right.\\
&\quad +3\,\frac{(1+\lambda_{HV})\,Q_u I_{3u}^2}{2s_w c_w^2} - \frac{(1+\lambda_{HV})\,I_{3u}^3}{2s_w^3 c_w^2}\\
&\quad + \frac{1}{2s_w}\left[\frac{1}{4m_W^2}\left(\sum_{j=1}^{3}\left(V_{ud_j}V^\dagger_{d_j u} m_{d_j}^2 Q_{d_j}\right) + \frac{m_u^2 Q_u\left(1+4I_{3u}^2\right)}{2}\right)\right.\\
&\quad \left.\left. + \sum_{j=1}^{3}\left(V_{ud_j}V^\dagger_{d_j u}\frac{(1+\lambda_{HV})}{2}\left(Q_{d_j} - \frac{c_w^2 + I_{3d_j}}{s_w^2}\right)\right)\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
Zd\bar{d} \quad : \quad C_- &= \frac{1}{8c_w}\left\{\frac{(1+\lambda_{HV})\,Q_d^3 s_w}{2c_w^2} + \frac{m_d^2}{8s_w m_W^2}\left[\sum_{i=1}^{3}\left(V_{u_i d}V^\dagger_{du_i}\left(Q_{u_i} - \frac{I_{3u_i}}{s_w^2}\right)\right)\right.\right.\\
&\quad \left.\left. + \left(Q_d - \frac{I_{3d}}{s_w^2}\right)\right]\right\}\\
C_+ &= \frac{1}{16c_w}\left\{(1+\lambda_{HV})\left(\frac{Q_d^3 s_w}{c_w^2} - \frac{Q_d^2 I_{3d}(1+2s_w^2)}{s_w c_w^2} + 3\frac{Q_d I_{3d}^2}{s_w c_w^2} - \frac{I_{3d}^3}{s_w^3 c_w^2}\right)\right.\\
&\quad + \frac{1}{s_w}\left[\frac{1}{4m_W^2}\left(\sum_{i=1}^{3}\left(V_{u_i d}V^\dagger_{du_i} m_{u_i}^2 Q_{u_i}\right) + \frac{m_d^2 Q_d\left(1+4I_{3d}^2\right)}{2}\right)\right.\\
&\quad \left.\left. + \sum_{i=1}^{3}\left(\frac{1+\lambda_{HV}}{2}\right)\left(V_{u_i d}V^\dagger_{du_i}\left(Q_{u_i} + \frac{c_w^2 - I_{3u_i}}{s_w^2}\right)\right)\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
Zl\bar{l} \quad : \quad C_- &= \frac{1}{8c_w}\left\{\frac{(1+\lambda_{HV})\,Q_l^3 s_w}{2c_w^2} + \frac{m_l^2}{4s_w m_W^2}\left[-\frac{1}{4s_w^2} + \frac{1}{2}\left(Q_l - \frac{I_{3l}}{s_w^2}\right)\right]\right\}\\
C_+ &= \frac{1}{16c_w}\left\{\left(\frac{Q_l^3 s_w}{c_w^2} - \frac{Q_l^2 I_{3l}(1+2s_w^2)}{s_w c_w^2}\right.\right.\\
&\quad \left. +3\,\frac{Q_l I_{3l}^2}{s_w c_w^2} - \frac{I_{3l}^3}{s_w^3 c_w^2}\right)(1+\lambda_{HV}) + \frac{1}{2s_w}\left[\frac{m_l^2 Q_l}{2m_W^2}\right.\\
&\quad \left.\left. + \frac{1}{s_w^2}(1+\lambda_{HV})\left(c_w^2 - I_{3\nu_l}\right)\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
Z\nu_l\bar{\nu}_l \quad : \quad C_- &= 0\\
C_+ &= \frac{1}{16c_w}\left\{-\frac{(1+\lambda_{HV})\,I_{3\nu_l}^3}{s_w^3 c_w^2} + \frac{1}{2s_w}\left[\frac{m_l^2 Q_l}{2m_W^2}\right.\right.\\
&\quad \left.\left. + (1+\lambda_{HV})\left(Q_l - \frac{c_w^2 + I_{3l}}{s_w^2}\right)\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
W^- u\bar{d} \quad : \quad C_- &= 0\\
C_+ &= \frac{V^\dagger_{du}}{16\sqrt{2}s_w}\left[\frac{Q_d I_{3u} + Q_u I_{3d} - Q_u Q_d}{c_w^2} - \frac{1}{s_w^2} + \frac{1}{4s_w^2 c_w^2}\right](1+\lambda_{HV})
\end{aligned}
$$

$$
W^+ d\bar{u} \quad : \quad C_- = 0
$$

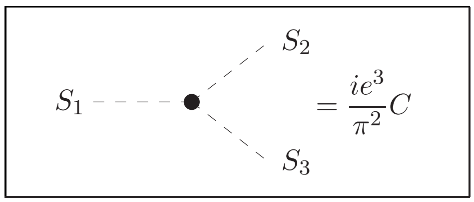
$$C_+ = \frac{V_{ud}}{16\sqrt{2}s_w}\left[\frac{Q_d I_{3u} + Q_u I_{3d} - Q_u Q_d}{c_w^2} - \frac{1}{s_w^2} + \frac{1}{4 s_w^2 c_w^2}\right](1+\lambda_{HV})$$

$$\left.\begin{array}{l} W^-\nu_l\bar{l} \\ W^+ l\bar{\nu}_l \end{array}\right\} \; : \; C_- = 0$$

$$C_+ = \frac{1}{16\sqrt{2}s_w}\left[\frac{Q_l I_{3\nu_l}}{c_w^2} - \frac{1}{s_w^2} + \frac{1}{4 s_w^2 c_w^2}\right](1+\lambda_{HV}) \tag{3.6}$$

3.2.3 Scalar-Scalar-Scalar effective vertices

The generic effective vertex is



with the actual values of S_1, S_2, S_3, and C

$$\left.\begin{array}{l} HH\chi \\ \chi\chi\chi \\ \chi\phi^+\phi^- \end{array}\right\} \; : \; C = 0$$

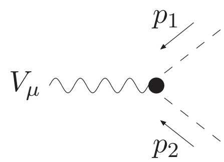
$$HHH \; : \; C = \frac{3}{32 s_w^3}\left[\frac{1-4\lambda_{HV}}{2}m_W + \frac{1}{m_W^3}\left(\sum_{i=1}^{6} m_{l_i}^4 + N_{\rm col}\sum_{i=1}^{6} m_{q_i}^4\right)\right.$$
$$\left. + \frac{1}{4}\left(1+\frac{1}{2c_w^2}\right)\frac{m_H^2}{m_W} + \frac{(1-4\lambda_{HV})\, m_W}{4c_w^4}\right]$$

$$H\chi\chi \; : \; C = \frac{1}{8 s_w^3}\left[\frac{1-4\lambda_{HV}}{8}m_W + \frac{1}{4m_W^3}\left(\sum_{i=1}^{6} m_{l_i}^4 + N_{\rm col}\sum_{i=1}^{6} m_{q_i}^4\right)\right.$$
$$\left. + \frac{1}{16}\left(1+\frac{1}{2c_w^2}\right)\frac{m_H^2}{m_W} + \frac{(1-4\lambda_{HV})\, m_W}{16c_w^4}\right]$$

$$H\phi^+\phi^- \; : \; C = \frac{1}{32 s_w^3}\left[\frac{1}{m_W^3}\left(\sum_{i=1}^{3} m_{e_i}^4 + N_{\rm col}\sum_{i,j=1}^{3}\left(V_{u_i d_j}V^\dagger_{d_j u_i}(m_{u_i}^4 + m_{d_j}^4)\right)\right)\right.$$
$$\left. + \frac{(1+2c_w^2)}{8c_w^2}\frac{m_H^2}{m_W} + \frac{3\,(1-4\lambda_{HV})}{4}m_W + \frac{1-4\lambda_{HV}}{4}\frac{s_w^2\left(1+c_w^2\right)}{c_w^4}m_W\right] \tag{3.7}$$

3.2.4 Vector-Scalar-Scalar effective vertices

The generic effective vertex is



$$= \frac{e^3}{\pi^2} C (p_1 - p_2)_\mu$$

with the actual values of V, S_1, S_2, and C

$$\left.\begin{array}{l} AHH \\ ZHH \\ A\chi\chi \\ Z\chi\chi \end{array}\right\} : \quad C = 0$$

$$A\chi H \quad : \quad C = \frac{5}{192 s_w^2}$$

$$Z\chi H \quad : \quad C = -\frac{1}{96 s_w c_w}\left[\frac{1+2c_w^2+20c_w^4}{8s_w^2 c_w^2} + \frac{1}{s_w^2 m_W^2}\left(\sum_{i=1}^{6}\left(m_{l_i}^2 + N_{\rm col} m_{q_i}^2\right)\right)\right]$$

$$\begin{aligned} A\phi^+\phi^- : \quad C = {} & \frac{i}{48 s_w^2}\left[\frac{1+12c_w^2}{8c_w^2} + \frac{1}{m_W^2}\left(-\sum_{i=1}^{3}\left(m_{e_i}^2 Q_{e_i}\right)\right.\right. \\ & \left.\left. + \; N_{\rm col}\sum_{i,j=1}^{3}\left(V_{u_i d_j} V_{d_j u_i}^\dagger (m_{u_i}^2 + m_{d_j}^2)\right)\right)\right] \end{aligned}$$

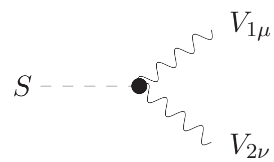
$$\begin{aligned} Z\phi^+\phi^- : \quad C = {} & \frac{i}{48 s_w c_w}\left\{\frac{1-24c_w^4}{16c_w^2 s_w^2} + \frac{1}{m_W^2}\left(-\sum_{i=1}^{3}\left(m_{e_i}^2\left(Q_{e_i} + \frac{I_{3\nu_i}}{s_w^2}\right)\right)\right.\right. \\ & \left.\left. + \; N_{\rm col}\sum_{i,j=1}^{3}\left(V_{u_i d_j} V_{d_j u_i}^\dagger\left[(m_{u_i}^2 + m_{d_j}^2) + \frac{m_{u_i}^2 I_{3d_j} - m_{d_i}^2 I_{3u_i}}{s_w^2}\right]\right)\right)\right\} \end{aligned}$$

$$\begin{aligned} \left.\begin{array}{l} W^+\phi^- H \\ W^- H\phi^+ \end{array}\right\} : \quad C = {} & \frac{i}{96 s_w^3}\left[\frac{1+22c_w^2}{8c_w^2} + \frac{1}{m_W^2}\left(\sum_{i=1}^{3} m_{e_i}^2\right.\right. \\ & \left.\left. + \; N_{\rm col}\sum_{i,j=1}^{3}\left(V_{u_i d_j} V_{d_j u_i}^\dagger (m_{u_i}^2 + m_{d_j}^2)\right)\right)\right] \end{aligned}$$

$$\begin{aligned} \left.\begin{array}{l} W^+\phi^- \chi \\ W^- \phi^+\chi \end{array}\right\} : \quad C = {} & \frac{1}{48 s_w^3}\left[-\frac{1+22c_w^2}{16c_w^2} + \frac{1}{m_W^2}\left(\sum_{i=1}^{3}\left(m_{e_i}^2 I_{3e_i}\right)\right.\right. \\ & \left.\left. - \; N_{\rm col}\sum_{i,j=1}^{3}\left(V_{u_i d_j} V_{d_j u_i}^\dagger (m_{u_i}^2 I_{3u_i} - m_{d_j}^2 I_{3d_j})\right)\right)\right] \end{aligned} \tag{3.8}$$

3.2.5 Scalar-Vector-Vector effective vertices

The generic effective vertex is



$$= \frac{ie^3}{\pi^2} C g_{\mu\nu}$$

with the actual values of S, V_1, V_2 and C

$$\left.\begin{array}{l} \chi AA \\ \chi AZ \\ \chi ZZ \\ \chi W^- W^+ \end{array}\right\} : \quad C = 0$$

$$\begin{aligned}
HAA \quad &: \quad C = -\frac{1}{8s_w}\left[\frac{1}{m_W}\left(\sum_{i=1}^{6}\left(m_{l_i}^2 Q_{l_i}^2\right) + N_{\rm col}\sum_{i=1}^{6}\left(m_{q_i}^2 Q_{q_i}^2\right)\right) + \frac{m_W}{2}\right] \\
HAZ \quad &: \quad C = \frac{1}{8c_w}\left\{\frac{1}{m_W}\left[\sum_{i=1}^{6}\left(m_{l_i}^2 Q_{l_i}\left(\frac{I_{3l_i}}{2s_w^2} - Q_{l_i}\right)\right)\right.\right. \\
&\qquad\quad \left.\left. +\ N_{\rm col}\sum_{i=1}^{6}\left(m_{q_i}^2 Q_{q_i}\left(\frac{I_{3q_i}}{2s_w^2} - Q_{q_i}\right)\right)\right] + \frac{m_W\left(1+2c_w^2\right)}{4s_w^2}\right\} \\
HZZ \quad &: \quad C = \frac{1}{8}\left\{\frac{1}{m_W c_w^2}\left[\sum_{i=1}^{6}\left(m_{l_i}^2\left(\frac{Q_{l_i} I_{3l_i}}{s_w} - Q_{l_i}^2 s_w - \frac{I_{3l_i}^2}{s_w^3}\right)\right)\right.\right. \\
&\qquad\quad \left.\left. +\ N_{\rm col}\sum_{i=1}^{6}\left(m_{q_i}^2\left(\frac{Q_{q_i} I_{3q_i}}{s_w} - Q_{q_i}^2 s_w - \frac{I_{3q_i}^2}{s_w^3}\right)\right)\right] + \frac{m_W\left(s_w^2-2\right)}{2s_w^3}\right\} \\
HW^-W^+ \quad &: \quad C = -\frac{1}{8s_w^3}\left[\frac{1}{4m_W}\left(\sum_{i=1}^{3} m_{e_i}^2\right.\right. \\
&\qquad\quad \left.\left. +\ N_{\rm col}\sum_{i,j=1}^{3}\left(V_{u_i d_j} V_{d_j u_i}^{\dagger}\left(m_{u_i}^2 + m_{d_j}^2\right)\right)\right) + m_W\right]
\end{aligned}$$

$$\left.\begin{array}{l} \phi^- A W^+ \\ \phi^+ W^- A \end{array}\right\} : \quad C = \frac{1}{32 s_w^2} K$$

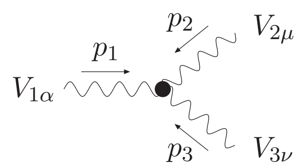
$$\left.\begin{array}{l} \phi^- Z W^+ \\ \phi^+ W^- Z \end{array}\right\} : \quad C = \frac{1}{32 s_w c_w} K \tag{3.9}$$

where

$$K = m_W + \frac{N_{\rm col}}{m_W}\sum_{i,j=1}^{3}\left(V_{u_i d_j} V_{d_j u_i}^{\dagger}\left(Q_{u_i} m_{d_j}^2 - Q_{d_j} m_{u_i}^2\right)\right) \tag{3.10}$$

3.2.6 Vector-Vector-Vector effective vertices

The generic effective vertex is



$$= \frac{ie^3}{\pi^2} C \left[g_{\alpha\mu}(p_2 - p_1)_\nu + g_{\mu\nu}(p_3 - p_2)_\alpha + g_{\nu\alpha}(p_1 - p_3)_\mu \right]$$

with the actual values of V_1, V_2, V_3 and C

$$\left.\begin{array}{c} AAA \\ AAZ \\ AZZ \\ ZZZ \end{array}\right\} : \quad C = 0$$

$$AW^+W^- \quad : \quad C = K$$

$$ZW^+W^- \quad : \quad C = -\frac{c_w}{s_w} K \tag{3.11}$$

where

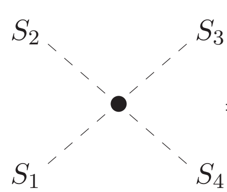
$$K = \frac{7 + 4\lambda_{HV}}{96 s_w^2} + \frac{1}{48 s_w^2} \left[\sum_{i=1}^{3} 1 + N_{\rm col} \sum_{i,j=1}^{3} \left(V_{u_i d_j} V^\dagger_{d_j u_i} \right) \right] \tag{3.12}$$

3.3 Electroweak effective vertices with 4 external legs

In this section, we give all possible contributing 4-point R_2 effective vertices.

3.3.1 Scalar-Scalar-Scalar-Scalar effective vertices

The generic effective vertex is



$$= \frac{ie^4}{\pi^2} C$$

with the actual values of S_1, S_2, S_3, S_4 and C

$$\left.\begin{array}{l} HHH\chi \\ H\chi\chi\chi \\ H\chi\phi^-\phi^+ \end{array}\right\} : \quad C = 0$$

$$\left.\begin{array}{l} HHHH \\ \chi\chi\chi\chi \end{array}\right\} : \quad C = \frac{1}{64 s_w^4} K_1$$

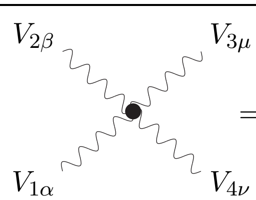
$$
\begin{aligned}
HH\chi\chi \quad &: \quad C = \frac{1}{192 s_w^4} K_1 \\
\left.\begin{array}{l} HH\phi^-\phi^+ \\ \chi\chi\phi^-\phi^+ \end{array}\right\} &: \quad C = \frac{1}{64 s_w^4} K_2 \\
\phi^-\phi^+\phi^-\phi^+ \quad &: \quad C = \frac{1}{32 s_w^4} K_3
\end{aligned}
\tag{3.13}
$$

where

$$
\begin{aligned}
K_1 &= \frac{1}{m_W^2}\left[\frac{5}{m_W^2}\sum_{i=1}^{6}\left(m_{l_i}^4 + N_{\rm col} m_{q_i}^4\right) + \frac{3}{2} m_H^2\left(1 + \frac{1}{2c_w^2}\right)\right] + \frac{1 - 12\lambda_{HV}}{2}\left(1 + \frac{1}{2c_w^4}\right) \\
K_2 &= \frac{1}{m_W^2}\left[\frac{5}{3m_W^2}\left(\sum_{i=1}^{3} m_{e_i}^4 + N_{\rm col}\sum_{i,j=1}^{3} V_{u_i d_j} V_{d_j u_i}^\dagger\left(m_{u_i}^4 + m_{d_j}^4\right)\right) + \frac{1}{2} m_H^2\left(1 + \frac{1}{2c_w^2}\right)\right] \\
&+ \frac{1 - 12\lambda_{HV}}{4}\left(1 + \frac{s_w^2}{3c_w^2}\left(1 + \frac{1}{c_w^2}\right)\right) \\
K_3 &= \frac{1}{m_W^2}\left[\frac{5}{3m_W^2}\left(\sum_{i=1}^{3} m_{e_i}^4 + N_{\rm col}\sum_{i,j,k,l=1}^{3}\left(V_{u_i d_j} V_{d_j u_k}^\dagger V_{u_k d_l} V_{d_l u_i}^\dagger\left(m_{u_i}^2 m_{u_k}^2 + m_{d_j}^2 m_{d_l}^2\right)\right)\right)\right. \\
&+ \left.\frac{1}{2} m_h^2\left(1 + \frac{1}{2c_w^2}\right)\right] + \left(\left(\frac{1}{4} - 3\lambda_{HV}\right)\left(1 + s_w^4\right)\right. \\
&+ \left.\left(\frac{1}{6} - 2\lambda_{HV}\right)\left(s_w^2 + \frac{2s_w^6}{c_w^2}\right) + \left(\frac{1}{12} - \lambda_{HV}\right)\frac{s_w^8}{c_w^4}\right)
\end{aligned}
\tag{3.14}
$$

3.3.2 Vector-Vector-Vector-Vector effective vertices

The generic effective vertex is



$$
= \frac{ie^4}{\pi^2}\left[C_1 g_{\alpha\beta} g_{\mu\nu} + C_2 g_{\alpha\mu} g_{\beta\nu} + C_3 g_{\alpha\nu} g_{\beta\mu}\right]
$$

with the actual values of V_1, V_2, V_3, V_4 C_1, C_2 and C_3

$$
\begin{aligned}
AAAA \quad : \quad C_1 &= \frac{1}{12}\left(-1 + \sum_{i=1}^{6} Q_{l_i}^4 + N_{\rm col}\sum_{i=1}^{6} Q_{q_i}^4\right) \\
C_2 &= C_1 \\
C_3 &= C_1
\end{aligned}
$$

$$
\begin{aligned}
AAAZ \quad : \quad C_1 &= \frac{1}{12}\left[\frac{c_w}{s_w} + \sum_{i=1}^{6}\left(\frac{s_w}{c_w}Q_{l_i}^4 - \frac{1}{2s_wc_w}Q_{l_i}^3 I_{3l_i}\right)\right. \\
&+ \left. N_{\rm col}\sum_{i=1}^{6}\left(\frac{s_w}{c_w}Q_{q_i}^4 - \frac{1}{2s_wc_w}Q_{q_i}^3 I_{3q_i}\right)\right] \\
C_2 &= C_1 \\
C_3 &= C_1
\end{aligned}
$$

$$
\begin{aligned}
AAZZ \quad : \quad C_1 &= \frac{1}{12}\left[-\frac{c_w^2}{s_w^2} + \frac{1}{2}\sum_{i=1}^{6}\left(\frac{s_w^2}{c_w^2}Q_{l_i}^4 + \left(\frac{s_w}{c_w}Q_{l_i}^2 - \frac{1}{s_wc_w}Q_{l_i} I_{3l_i}\right)^2\right)\right. \\
&+ \left. \frac{N_{\rm col}}{2}\sum_{i=1}^{6}\left(\frac{s_w^2}{c_w^2}Q_{q_i}^4 + \left(\frac{s_w}{c_w}Q_{q_i}^2 - \frac{1}{s_wc_w}Q_{q_i} I_{3q_i}\right)^2\right)\right] \\
C_2 &= C_1 \\
C_3 &= C_1
\end{aligned}
$$

$$
\begin{aligned}
AZZZ \quad : \quad C_1 &= \frac{1}{12}\left[\frac{c_w^3}{s_w^3} + \sum_{i=1}^{6}\left(\frac{s_w^3}{c_w^3}Q_{l_i}^4 - \frac{3}{2}\frac{s_w}{c_w^3}Q_{l_i}^3 I_{3l_i}\right.\right. \\
&+ \left. \frac{3}{2}\frac{1}{s_wc_w^3}Q_{l_i}^2 I_{3l_i}^2 - \frac{1}{2s_w^3c_w^3}Q_{l_i} I_{3l_i}^3\right) \\
&+ N_{\rm col}\sum_{i=1}^{6}\left(\frac{s_w^3}{c_w^3}Q_{q_i}^4 - \frac{3}{2}\frac{s_w}{c_w^3}Q_{q_i}^3 I_{3q_i}\right. \\
&+ \left.\left. \frac{3}{2}\frac{1}{s_wc_w^3}Q_{q_i}^2 I_{3q_i}^2 - \frac{1}{2s_w^3c_w^3}Q_{q_i} I_{3q_i}^3\right)\right] \\
C_2 &= C_1 \\
C_3 &= C_1
\end{aligned}
$$

$$
\begin{aligned}
ZZZZ \quad : \quad C_1 &= \frac{1}{12}\left[-\frac{c_w^4}{s_w^4} + \sum_{i=1}^{6}\left(\frac{s_w^4}{c_w^4}Q_{l_i}^4 - 2\frac{s_w^2}{c_w^4}Q_{l_i}^3 I_{3l_i}\right.\right. \\
&+ \left. \frac{3}{c_w^4}Q_{l_i}^2 I_{3l_i}^2 - \frac{2}{s_w^2c_w^4}Q_{l_i} I_{3l_i}^3 + \frac{1}{2s_w^4c_w^4}I_{3l_i}^4\right) \\
&+ N_{\rm col}\sum_{i=1}^{6}\left(\frac{s_w^4}{c_w^4}Q_{q_i}^4 - 2\frac{s_w^2}{c_w^4}Q_{q_i}^3 I_{3q_i}\right. \\
&+ \left.\left. \frac{3}{c_w^4}Q_{q_i}^2 I_{3q_i}^2 - \frac{2}{s_w^2c_w^4}Q_{q_i} I_{3q_i}^3 + \frac{1}{2s_w^4c_w^4}I_{3q_i}^4\right)\right] \\
C_2 &= C_1 \\
C_3 &= C_1
\end{aligned}
$$

$$
\begin{aligned}
AAW^-W^+ \quad : \quad C_1 &= \frac{1}{16s_w^2}\left[\frac{10+4\lambda_{HV}}{3}+\sum_{i=1}^{3}1+\frac{25}{27}N_{\rm col}\sum_{i,j=1}^{3}\left(V_{u_id_j}V^\dagger_{d_ju_i}\right)\right]\\
C_2 &= -\frac{1}{16s_w^2}\left[\frac{7+2\lambda_{HV}}{3}+\frac{1}{3}\sum_{i=1}^{3}1+\frac{11}{27}N_{\rm col}\sum_{i,j=1}^{3}\left(V_{u_id_j}V^\dagger_{d_ju_i}\right)\right]\\
C_3 &= C_2
\end{aligned}
$$

$$
\begin{aligned}
AZW^-W^+ \quad : \quad C_1 &= \frac{1}{16s_wc_w}\left[-\frac{(10+4\lambda_{HV})\,c_w^2}{3s_w^2}+\left(1-\frac{11}{12s_w^2}\right)\sum_{i=1}^{3}1\right.\\
&\quad\left. +\ N_{\rm col}\sum_{i,j=1}^{3}\left(V_{u_id_j}V^\dagger_{d_ju_i}\left(\frac{25}{27}-\frac{11}{12s_w^2}\right)\right)\right]\\
C_2 &= \frac{1}{16s_wc_w}\left[\frac{7+2\lambda_{HV}}{3}\frac{c_w^2}{s_w^2}+\left(\frac{5}{12s_w^2}-\frac{1}{3}\right)\sum_{i=1}^{3}1\right.\\
&\quad\left. +\ N_{\rm col}\sum_{i,j=1}^{3}\left(V_{u_id_j}V^\dagger_{d_ju_i}\left(\frac{5}{12s_w^2}-\frac{11}{27}\right)\right)\right]\\
C_3 &= C_2
\end{aligned}
$$

$$
\begin{aligned}
ZZW^-W^+ \quad : \quad C_1 &= \frac{(5+2\lambda_{HV})\,c_w^2}{24s_w^4}+\frac{1}{16c_w^2}\left[\left(1-\frac{11}{6s_w^2}+\frac{11}{12s_w^4}\right)\sum_{i=1}^{3}1\right.\\
&\quad\left. +\ N_{\rm col}\left(\frac{25}{27}-\frac{11}{6s_w^2}+\frac{11}{12s_w^4}\right)\sum_{i,j=1}^{3}\left(V_{u_id_j}V^\dagger_{d_ju_i}\right)\right]\\
C_2 &= -\frac{(7+2\lambda_{HV})\,c_w^2}{48s_w^4}+\frac{1}{16c_w^2}\left[\left(-\frac{1}{3}+\frac{5}{6s_w^2}-\frac{5}{12s_w^4}\right)\sum_{i=1}^{3}1\right.\\
&\quad\left. +\ N_{\rm col}\left(-\frac{11}{27}+\frac{5}{6s_w^2}-\frac{5}{12s_w^4}\right)\sum_{i,j=1}^{3}\left(V_{u_id_j}V^\dagger_{d_ju_i}\right)\right]\\
C_3 &= C_2
\end{aligned}
$$

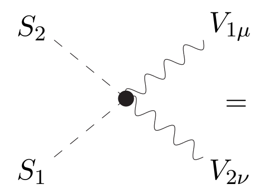
$$
\begin{aligned}
W^-W^+W^-W^+ \quad : \quad C_1 &= \frac{1}{16s_w^4}\left[\frac{3+2\lambda_{HV}}{3}+\frac{1}{2}\sum_{i=1}^{3}1\right.\\
&\quad\left. +\ \frac{N_{\rm col}}{2}\sum_{i,j,k,m=1}^{3}\left(V_{u_id_j}V^\dagger_{d_ju_k}V_{u_kd_m}V^\dagger_{d_mu_i}\right)\right]\\
C_2 &= -\frac{1}{8s_w^4}\left[\frac{7+2\lambda_{HV}}{3}+\frac{5}{12}\sum_{i=1}^{3}1\right.
\end{aligned}
$$

$$
+\ \frac{5}{12} N_{\rm col} \sum_{i,j,k,m=1}^{3} \left(V_{u_i d_j} V^{\dagger}_{d_j u_k} V_{u_k d_m} V^{\dagger}_{d_m u_i} \right) \Bigg]
$$

$$
C_3 = C_1 \tag{3.15}
$$

3.3.3 Scalar-Scalar-Vector-Vector effective vertices

The generic effective vertex is



$$
= \frac{ie^4}{\pi^2} C g_{\mu\nu}
$$

with the actual values of S_1, S_2, V_1, V_2 and C

$$
\left.\begin{array}{l} H\chi AA \\ H\chi AZ \\ H\chi ZZ \\ H\chi W^+W^- \end{array}\right\} : \quad C = 0
$$

$$
\left.\begin{array}{l} HHAA \\ \chi\chi AA \end{array}\right\} : \quad C = \frac{1}{16 s_w^2} \left\{ \frac{1}{12} - \frac{1}{m_W^2} \left[\sum_{i=1}^{6} \left(Q_{l_i}^2 m_{l_i}^2 \right) + N_{\rm col} \sum_{i=1}^{6} \left(Q_{q_i}^2 m_{q_i}^2 \right) \right] \right\}
$$

$$
\begin{aligned}
\left.\begin{array}{l} HHAZ \\ \chi\chi AZ \end{array}\right\} : \quad C = \frac{1}{16 s_w} \Bigg\{ & \frac{4 + s_w^2}{12 s_w^2 c_w} + \frac{1}{m_W^2 c_w} \left[\sum_{i=1}^{6} \left(Q_{l_i} m_{l_i}^2 \left(\frac{I_{3l_i}}{2 s_w^2} - Q_{l_i} \right) \right) \right. \\
& \left. + \ N_{\rm col} \sum_{i=1}^{6} \left(Q_{q_i} m_{q_i}^2 \left(\frac{I_{3q_i}}{2 s_w^2} - Q_{q_i} \right) \right) \right] \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\left.\begin{array}{l} HHZZ \\ \chi\chi ZZ \end{array}\right\} : \quad C = -\frac{1}{16 c_w^2} \Bigg\{ & \frac{1 + 2c_w^2 + 40 c_w^4 - 4 c_w^6}{48 s_w^4 c_w^2} \\
& + \ \frac{1}{m_W^2} \left[\sum_{i=1}^{6} \left(m_{l_i}^2 \left(Q_{l_i}^2 + \frac{4 I_{3l_i}^2}{3 s_w^4} - \frac{Q_{l_i} I_{3l_i}}{s_w^2} \right) \right) \right. \\
& \left. + \ N_{\rm col} \sum_{i=1}^{6} \left(m_{q_i}^2 \left(Q_{q_i}^2 + \frac{4 I_{3q_i}^2}{3 s_w^4} - \frac{Q_{q_i} I_{3q_i}}{s_w^2} \right) \right) \right] \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\left.\begin{array}{l} HHW^-W^+ \\ \chi\chi W^-W^+ \end{array}\right\} : \quad C = -\frac{1}{48 s_w^4} \Bigg\{ & \frac{1 + 38 c_w^2}{16 c_w^2} \\
& + \ \frac{1}{m_W^2} \left[\sum_{i=1}^{3} m_{e_i}^2 + N_{\rm col} \sum_{i,j=1}^{3} \left(V_{u_i d_j} V^{\dagger}_{d_j u_i} \left(m_{u_i}^2 + m_{d_j}^2 \right) \right) \right] \Bigg\}
\end{aligned}
$$

$$
\left.\begin{array}{l} H\phi^+W^-A \\ \phi^- HAW^+ \end{array}\right\} : \quad C = K_1
$$

$$\chi\phi^+W^-A \quad : \quad C = -iK_1$$

$$\phi^-\chi AW^+ \quad : \quad C = iK_1$$

$$\left.\begin{array}{l} H\phi^+W^-Z \\ \phi^-HZW^+ \end{array}\right\} : \quad C = K_2$$

$$\chi\phi^+W^-Z \quad : \quad C = -iK_2$$

$$\phi^-\chi ZW^+ \quad : \quad C = iK_2$$

$$\begin{aligned}
\phi^-\phi^+AA \quad : \quad C &= -\frac{1}{12s_w^2}\left\{\frac{1+21c_w^2}{16c_w^2} + \frac{1}{m_W^2}\left[\sum_{i=1}^{3} m_{e_i}^2\right.\right. \\
&+ \left.\left.\frac{5}{6}N_{\mathrm{col}}\sum_{i,j=1}^{3}\left(V_{u_id_j}V^\dagger_{d_ju_i}(m_{u_i}^2+m_{d_j}^2)\right)\right]\right\}
\end{aligned}$$

$$\begin{aligned}
\phi^-\phi^+AZ \quad : \quad C &= \frac{1}{12s_wc_w}\left\{\frac{42c_w^4-10c_w^2-1}{32s_w^2c_w^2}\right. \\
&- \frac{1}{m_W^2}\left[\sum_{i=1}^{3}\left(m_{e_i}^2Q_{e_i}\left(Q_{e_i}+\frac{5}{8}\frac{I_{3\nu_i}}{s_w^2}\right)\right)\right. \\
&+ N_{\mathrm{col}}\sum_{i,j=1}^{3}\left[V_{u_id_j}V^\dagger_{d_ju_i}\left(m_{u_i}^2\left(\frac{5}{6}-\frac{I_{3d_i}}{s_w^2}\left(Q_{d_j}-\frac{5}{8}Q_{u_i}\right)\right)\right.\right. \\
&+ \left.\left.\left.\left.m_{d_j}^2\left(\frac{5}{6}-\frac{I_{3u_i}}{s_w^2}\left(Q_{u_i}-\frac{5}{8}Q_{d_j}\right)\right)\right)\right]\right]\right\}
\end{aligned}$$

$$\begin{aligned}
\phi^-\phi^+ZZ \quad : \quad C &= \frac{1}{12c_w^2}\left\{\frac{-1+2c_w^2+44c_w^4-84c_w^6}{64s_w^4c_w^2}\right. \\
&- \frac{1}{m_W^2}\left[\sum_{i=1}^{3}\left(m_{e_i}^2\left(Q_{e_i}^2+\frac{5}{4}\frac{Q_{e_i}I_{3\nu_i}}{s_w^2}+\frac{I_{3\nu_i}^2}{s_w^4}\right)\right)\right. \\
&+ N_{\mathrm{col}}\sum_{i,j=1}^{3}\left[V_{u_id_j}V^\dagger_{d_ju_i}\left(m_{u_i}^2\left(\frac{5}{6}-\frac{I_{3d_i}}{s_w^2}\left(2Q_{d_j}-\frac{5}{4}Q_{u_i}\right)+\frac{I_{3d_i}^2}{s_w^4}\right)\right.\right. \\
&+ \left.\left.\left.\left.m_{d_j}^2\left(\frac{5}{6}-\frac{I_{3u_i}}{s_w^2}\left(2Q_{u_i}-\frac{5}{4}Q_{d_j}\right)+\frac{I_{3u_i}^2}{s_w^4}\right)\right)\right]\right]\right\}
\end{aligned}$$

$$\phi^-\phi^+W^-W^+ \quad : \quad C = -\frac{1}{48s_w^4}\left\{\frac{1}{m_W^2}\left[\left(\sum_{i=1}^{3}m_{e_i}^2\right.\right.\right.$$

$$
+ \; N_{\rm col} \sum_{i,j,k,l=1}^{3} \left(V_{u_i d_j} V^{\dagger}_{d_j u_k} V_{u_k d_l} V^{\dagger}_{d_l u_i} \left(m_{u_i} m_{u_k} + m_{d_j} m_{d_l} \right) \right) \Bigg) \Bigg]
$$

$$
+ \; \frac{38 c_w^2 + 1}{16 c_w^2} \Bigg\}
\tag{3.16}
$$

with

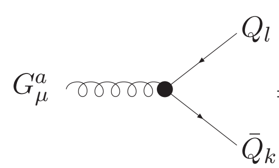
$$
\begin{aligned}
K_1 &= \frac{1}{24 s_w^3} \left\{ \frac{1 + 22 c_w^2}{32 c_w^2} + K \right\} \\
K_2 &= \frac{1}{24 s_w^2 c_w} \left\{ \frac{1 + 21 c_w^2 - 22 c_w^4}{32 c_w^2 s_w^2} + K \right\} \\
K &= \frac{1}{8 m_W^2} \left[\sum_{i=1}^{3} m_{e_i}^2 + N_{\rm col} \sum_{i,j=1}^{3} \left(V_{u_i d_j} V^{\dagger}_{d_j u_i} \left(3 m_{d_j}^2 + 2 m_{u_i}^2 \right) \right) \right]
\end{aligned}
\tag{3.17}
$$

3.4 Mixed Electroweak/QCD corrections

In [13], all mixed R_2 QCD/Electroweak vertices with internal QCD particle and external weak fields are presented. For completeness, we give here the only contributing Mixed Electroweak/QCD R_2 effective vertex, with internal EW particles and external colored states.

3.4.1 Gluon-Quark-Quark effective vertex

The generic effective vertex is



$$
= \frac{i g_s e^2}{\pi^2} t^a_{kl} (C_- \Omega^- + C_+ \Omega^+) \gamma_\mu
$$

with the actual values of Q, $\bar{Q}$, C_- and C_+

$$
\begin{aligned}
u\bar{u} \;\; : \;\; C_- &= \frac{1}{16} \left[(1 + \lambda_{HV}) \frac{Q_u^2}{c_w^2} + \frac{m_u^2}{2 s_w^2 m_W^2} \left(\frac{1}{2} \sum_{j=1}^{3} \left(V_{u d_j} V^{\dagger}_{d_j u} \right) + \frac{1}{4} + I_{3u}^2 \right) \right] \\
C_+ &= \frac{1}{16} \Bigg[(1 + \lambda_{HV}) \left(\frac{1}{c_w^2} \left(Q_u^2 + \frac{I_{3u}^2}{s_w^2} - 2 Q_u I_{3u} \right) + \frac{1}{2 s_w^2} \sum_{j=1}^{3} \left(V_{u d_j} V^{\dagger}_{d_j u} \right) \right) \\
&\quad + \frac{1}{2 m_W^2 s_w^2} \left(\frac{1}{2} \sum_{j=1}^{3} \left(V_{u d_j} V^{\dagger}_{d_j u} m_{d_j}^2 \right) + m_u^2 \left(\frac{1}{4} + I_{3u}^2 \right) \right) \Bigg] \\
d\bar{d} \;\; : \;\; C_- &= \frac{1}{16} \left[(1 + \lambda_{HV}) \frac{Q_d^2}{c_w^2} + \frac{m_d^2}{2 s_w^2 m_W^2} \left(\frac{1}{2} \sum_{i=1}^{3} \left(V_{u_i d} V^{\dagger}_{d u_i} \right) + \frac{1}{4} + I_{3d}^2 \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
C_+ = \frac{1}{16}\Bigg[&(1+\lambda_{HV})\left(\frac{1}{c_w^2}\left(Q_d^2+\frac{I_{3d}^2}{s_w^2}-2Q_dI_{3d}\right)+\frac{1}{2s_w^2}\sum_{i=1}^{3}\left(V_{u_id}V^\dagger_{du_i}\right)\right)\\
&+\frac{1}{2m_W^2s_w^2}\left(\frac{1}{2}\sum_{i=1}^{3}\left(V_{u_id}V^\dagger_{du_i}m_{u_i}^2\right)+m_d^2\left(\frac{1}{4}+I_{3d}^2\right)\right)\Bigg]
\end{aligned}
\tag{3.18}
$$

4. Tests and findings

We performed several checks on our formulae. First of all, we derived them by means of two independent calculations, secondly, we explicitly checked the gauge invariance of our results with the help of the Ward Identities listed in app. A, that we derived, by using the Background Field Method described in [20], in the way we detail in the appendix. Given the fact that only $\mathrm{R} = \mathrm{R}_1 + \mathrm{R}_2$ is gauge invariant, we adopted the following strategy. The terms proportional to λ_{HV} in our effective vertices are expected to be gauge invariant by themselves. Such terms can only be generated by R_2, so that we could explicitly check, by using `FORM`, that this part of our results fulfills all of the Ward identities of app. A, both in the 't Hooft-Feynman gauge and in the Background Field Method approach. This provides an explicit test of the gauge invariance of the Four Dimensional Helicity regularization scheme in the complete Standard Model at 1-loop, and we consider this result as a by-product of our calculation.

To also test the parts not proportional to λ_{HV}, we computed analytically R_1 [4], we added it to R_2 and checked that the quantity $\mathrm{R}_1 + \mathrm{R}_2$ fulfills all of the 2-point and 3-point Ward identities listed in the appendix. In the 4-point case, many new vertices are present in R_1 that do not contribute to R_2, such as VVVS, and, given the fact that, after all, we just need to check R_2, we limited ourselves to verify the first six 4-point Ward identities given in app. A.6, which are the only ones including both the VVVV and VVV vertices, but not VVVS. The described gauge invariance test on $\mathrm{R}_1 + \mathrm{R}_2$ is a very powerful and non trivial one. In fact, the analytic expressions for R_1 are, in general, much more complicated than the ones for R_2, involving a huge amount of terms with different combinations/powers of Gram determinants.

5. Conclusions

In the last few years, new techniques have been developed to efficiently deal with the problem of computing the radiative corrections needed to cope with the complicated phenomenology expected at LHC and ILC. Nowadays, thanks to the OPP technique, the so called Cut Constructible part of the virtual 1-loop amplitudes can be obtained, in a purely numerical way, by means of a calculation of the same conceptual complexity of a tree level one. However, the determination of the remaining rational part R of the amplitude requires a different strategy. In the treatment at the *integrand level*, that we follow in this paper, a piece of R, called R_1, can be directly linked to the Cut Constructible part of the

[4] We extracted the R_1 part of the contributing tensor integrals by using the Passarino-Veltman [21] reduction technique and by further checking numerically the expressions with the help of CutTools [16].

amplitude, and it is therefore numerically and automatically produced, in the OPP framework, by codes like CutTools. The remaining part of R, called R_2, cannot be determined numerically in 4 dimensions, and requires an explicit computation in terms of the vertices of the theory at hand, up to four external legs. From the knowledge of these vertices, a finite set of effective tree level Feynman rules can be extracted to be used to compute R_2 for processes with an arbitrary number of external legs. Such effective R_2 Feynman rules have been already given, in the literature, for QED and QCD and, in this paper, we completed the list by computing and presenting the set of R_2 Feynman rules for the Electroweak sector, which was the last missing piece for completely automatizing, in the framework of the OPP method, the 1-loop calculations in the $SU(3) \times SU(2) \times U(1)$ Standard Model.

In addition, since R_2 is the only part of the amplitude sensitive to the choice of the regularization scheme, we explicitly proved, by checking a large set of Ward identities, the gauge invariance of the Four Dimensional Helicity regularization scheme in the full Electroweak sector at 1-loop.

Acknowledgments

Many thanks to Huasheng Shao for having recomputed, independently of us, all of the R_2 effective vertices. Thanks to his help and by comparing with an independent computation we performed in a general R_ξ gauge, we could fix, in the present version, a few problems in our formulae. We would like to thank Costas Papadopoulos, Ronald Kleiss and Stefan Dittmaier for useful discussions. R.P. is also indebted with Mauro Moretti and Fulvio Piccinini for discussing the gauge invariance arguments presented in section 2. R.P.'s and I.M.'s research was partially supported by the RTN European Programme MRTN-CT-2006-035505 (HEPTOOLS, Tools and Precision Calculations for Physics Discoveries at Colliders). M.V.G.'s research was supported by INFN. The research of R.P. and M.V.G. was also supported by the MEC project FPA2008-02984.

Appendices

A. Ward identities

The Background Field Method (BFM) is a technique for quantizing gauge theories without losing explicit gauge invariance of the effective action [20, 22, 23, 24, 25]. Starting from a classical Lagrangian, one can achieve this by decomposing the usual fields into background fields and quantum fields. Then, the background fields are treated as external sources, while the quantum fields are variables of integration in the functional integral. A gauge fixing term is added, which only breaks the invariance with respect to the quantum gauge transformations, while the invariance with respect to background-field gauge transformations is preserved. From the Lagrangian mentioned above, one can construct an effective action $\Gamma[\hat{V}, \hat{S}, F, \bar{F}]$, where $\hat{V}$ refers to the background gauge fields, $\hat{S}$ to the background scalar fields and $F, \bar{F}$ to the fermion fields (for all fields that do not enter the gauge-fixing term, quantization is identical in the BFM and in the conventional formalism. Their Feynman rules for the background fields and quantum fields are also identical, so there is no

need to distinguish them). This effective action is invariant under the background gauge transformations given in eqs. 21, 22 of [20]. This invariance implies that

$$\frac{\delta\Gamma}{\delta\hat{\theta}^a} = 0\,, \tag{A.1}$$

where $a = A, Z, W^\pm$ and $\hat{\theta}^a$ are the infinitesimal gauge transformations of the background fields. By combining these formulas with eqs. 21, 22 of [20], one can produce eqs. 4, 5 and 6 of [24]. By differentiating them with respect to background fields and setting the fields equal to zero, one obtains Ward identities for the vertex functions that are precisely the Ward identities related to the classical Lagrangian. In the papers [20] and [24] some of these Ward identities are listed (see also [26]). In the following, we extend this list by producing more Ward identities useful for our checks [5].

A.1 Ward identities involving VV, VS and SS

$$k^\mu \Gamma^{AA}_{\mu\nu}(k,-k) = k^\mu \Gamma^{AZ}_{\mu\nu}(k,-k) = 0 \tag{A.2}$$

$$k^\mu \Gamma^{ZZ}_{\mu\nu}(k,-k) - iM_Z \Gamma^{\chi Z}_{\nu}(k,-k) = 0 \tag{A.3}$$

$$k^\mu \Gamma^{W^\pm W^\mp}_{\mu\nu}(k,-k) \mp M_W \Gamma^{\phi^\pm W^\mp}_{\nu}(k,-k) = 0 \tag{A.4}$$

$$k^\mu \Gamma^{Z\chi}_{\mu\nu}(k,-k) - iM_Z \Gamma^{\chi\chi}_{\nu}(k,-k) + \frac{ie}{2c_w s_w} T^H = 0 \tag{A.5}$$

$$k^\mu \Gamma^{W^\pm \phi^\mp}_{\mu\nu}(k,-k) \mp M_W \Gamma^{\phi^\pm \phi^\mp}_{\nu}(k,-k) \pm \frac{e}{2s_w} T^H = 0 \tag{A.6}$$

In the previous identities, T^H is the Higgs tadpole contribution. We have found a non-vanishing R_2 contribution to T^H, due to the coupling of H with Z and W, while R_1 does not contribute to T^H.

A.2 Ward identities involving VFF, SFF and FF

$$k^\mu \Gamma^{A\bar{f}f}_{\mu}(k,\bar{p},p) + eQ_f(\Gamma^{\bar{f}f}(\bar{p},k+p) - \Gamma^{\bar{f}f}(k+\bar{p},p)) = 0 \tag{A.7}$$

$$\begin{aligned} k^\mu \Gamma^{Z\bar{f}f}_{\mu}(k,\bar{p},p) - iM_Z \Gamma^{\chi\bar{f}f}(k,\bar{p},p) - e(\Gamma^{\bar{f}f}(\bar{p},k+p)(v_f - a_f\gamma_5) \\ -(v_f + a_f\gamma_5)\Gamma^{\bar{f}f}(k+\bar{p},p)) = 0 \end{aligned} \tag{A.8}$$

[5] We assume $V_{ud} = V^\dagger_{du} = 1$ and understand a sum over colors.

$$k^\mu \Gamma_\mu^{W^+\bar f_u f_d}(k,\bar p,p) - M_W \Gamma^{\phi^+\bar f_u f_d}(k,\bar p,p) - \frac{e}{\sqrt{2}s_w}(\Gamma^{\bar f_u f_u}(\bar p,k+p)\Omega_- - \Omega_+\Gamma^{\bar f_d f_d}(k+\bar p,p)) = 0 \quad (A.9)$$

$$k^\mu \Gamma_\mu^{W^-\bar f_d f_u}(k,\bar p,p) + M_W \Gamma^{\phi^-\bar f_d f_u}(k,\bar p,p) - \frac{e}{\sqrt{2}s_w}(\Gamma^{\bar f_d f_d}(\bar p,k+p)\Omega_- - \Omega_+\Gamma^{\bar f_u f_u}(k+\bar p,p)) = 0 \quad (A.10)$$

In the previous expressions, f_u is a fermion with I_{3f}=1/2, f_d is the fermion of the same weak-isospin doublet with I_{3f}= -1/2, $v_f = (I_{3f} - 2s_w^2 Q_f)/(2s_w c_w)$ and $a_f = I_{3f}/(2s_w c_w)$.

A.3 Ward identities involving VVV, VVS and VV

$$k^\mu \Gamma_{\mu\nu\sigma}^{AW^+W^-}(k,k_+,k_-) - e(\Gamma_{\nu\sigma}^{W^+W^-}(k_+,k+k_-) - \Gamma_{\nu\sigma}^{W^+W^-}(k+k_+,k_-)) = 0 \quad (A.11)$$

$$k_+^\mu \Gamma_{\mu\nu\sigma}^{W^+W^-A}(k_+,k_-,k) - M_W \Gamma_{\nu\sigma}^{\phi^+W^-A}(k_+,k_-,k) - e\Gamma_{\sigma\nu}^{W^+W^-}(k+k_+,k_-) + e(\Gamma_{\sigma\nu}^{AA}(k,k_++k_-) - \frac{c_w}{s_w}\Gamma_{\sigma\nu}^{AZ}(k,k_++k_-)) = 0 \quad (A.12)$$

$$k_-^\mu \Gamma_{\mu\nu\sigma}^{W^-W^+A}(k_-,k_+,k) + M_W \Gamma_{\nu\sigma}^{\phi^-W^+A}(k_-,k_+,k) + e\Gamma_{\sigma\nu}^{W^-W^+}(k+k_-,k_+) - e(\Gamma_{\sigma\nu}^{AA}(k,k_++k_-) - \frac{c_w}{s_w}\Gamma_{\sigma\nu}^{AZ}(k,k_++k_-)) = 0 \quad (A.13)$$

$$k^\mu \Gamma_{\mu\nu\sigma}^{ZW^+W^-}(k,k_+,k_-) - iM_Z \Gamma_{\nu\sigma}^{\chi W^+W^-}(k,k_+,k_-) - e\frac{c_w}{s_w}(\Gamma_{\nu\sigma}^{W^+W^-}(k+k_+,k_-) - \Gamma_{\sigma\nu}^{W^-W^+}(k+k_-,k_+)) = 0 \quad (A.14)$$

$$k_+^\mu \Gamma_{\mu\nu\sigma}^{W^+W^-Z}(k_+,k_-,k) - M_W \Gamma_{\nu\sigma}^{\phi^+W^-Z}(k_+,k_-,k) + e\frac{c_w}{s_w}\Gamma_{\sigma\nu}^{W^+W^-}(k+k_+,k_-) + e(\Gamma_{\nu\sigma}^{AZ}(k_++k_-,k) - \frac{c_w}{s_w}\Gamma_{\nu\sigma}^{ZZ}(k_++k_-,k)) = 0 \quad (A.15)$$

$$k_-^\mu \Gamma_{\mu\nu\sigma}^{W^-W^+Z}(k_-,k_+,k) + M_W \Gamma_{\nu\sigma}^{\phi^-W^+Z}(k_-,k_+,k) - e\frac{c_w}{s_w}\Gamma_{\sigma\nu}^{W^-W^+}(k+k_-,k_+) - e(\Gamma_{\nu\sigma}^{AZ}(k_++k_-,k) - \frac{c_w}{s_w}\Gamma_{\nu\sigma}^{ZZ}(k_++k_-,k)) = 0 \quad (A.16)$$

A.4 Ward identities involving VVS, VSS and VS

$$
\begin{aligned}
&k_1^\mu \Gamma^{AAH}_{\mu\nu}(k_1,k_2,k_3) = k_1^\mu \Gamma^{AA\chi}_{\mu\nu}(k_1,k_2,k_3) \\
&= k_1^\mu \Gamma^{AZH}_{\mu\nu}(k_1,k_2,k_3) = k_1^\mu \Gamma^{AZ\chi}_{\mu\nu}(k_1,k_2,k_3) = 0
\end{aligned} \tag{A.17}
$$

$$
k^\mu \Gamma^{AW^+\phi^-}_{\mu\nu}(k,k_+,k_-) + e\Gamma^{W^+\phi^-}_\nu(k+k_+,k_-) - e\Gamma^{\phi^-W^+}_\nu(k+k_-,k_+) = 0 \tag{A.18}
$$

$$
k^\mu \Gamma^{AW^-\phi^+}_{\mu\nu}(k,k_-,k_+) - e\Gamma^{W^-\phi^+}_\nu(k+k_-,k_+) + e\Gamma^{\phi^+W^-}_\nu(k+k_+,k_-) = 0 \tag{A.19}
$$

$$
k_1^\mu \Gamma^{ZAH}_{\mu\nu}(k_1,k_2,k_3) - iM_Z\Gamma^{\chi AH}_\nu(k_1,k_2,k_3) - \frac{ie}{2c_ws_w}\Gamma^{\chi A}_\nu(k_1+k_3,k_2) = 0 \tag{A.20}
$$

$$
k_1^\mu \Gamma^{ZA\chi}_{\mu\nu}(k_1,k_2,k_3) - iM_Z\Gamma^{\chi A\chi}_\nu(k_1,k_2,k_3) + \frac{ie}{2c_ws_w}\Gamma^{HA}_\nu(k_1+k_3,k_2) = 0 \tag{A.21}
$$

$$
k_1^\mu \Gamma^{ZZH}_{\mu\nu}(k_1,k_2,k_3) - iM_Z\Gamma^{\chi ZH}_\nu(k_1,k_2,k_3) - \frac{ie}{2c_ws_w}\Gamma^{\chi Z}_\nu(k_1+k_3,k_2) = 0 \tag{A.22}
$$

$$
k_1^\mu \Gamma^{ZZ\chi}_{\mu\nu}(k_1,k_2,k_3) - iM_Z\Gamma^{\chi Z\chi}_\nu(k_1,k_2,k_3) + \frac{ie}{2c_ws_w}\Gamma^{HZ}_\nu(k_1+k_3,k_2) = 0 \tag{A.23}
$$

$$
\begin{aligned}
&k^\mu \Gamma^{ZW^+\phi^-}_{\mu\nu}(k,k_+,k_-) - iM_Z\Gamma^{\chi W^+\phi^-}_\nu(k,k_+,k_-) \\
&-e\frac{c_w}{s_w}\Gamma^{W^+\phi^-}_\nu(k+k_+,k_-) + e\frac{c_w^2-s_w^2}{2c_ws_w}\Gamma^{\phi^-W^+}_\nu(k+k_-,k_+) = 0
\end{aligned} \tag{A.24}
$$

$$
\begin{aligned}
&k^\mu \Gamma^{ZW^-\phi^+}_{\mu\nu}(k,k_-,k_+) - iM_Z\Gamma^{\chi W^-\phi^+}_\nu(k,k_-,k_+) \\
&+e\frac{c_w}{s_w}\Gamma^{W^-\phi^+}_\nu(k+k_-,k_+) - e\frac{c_w^2-s_w^2}{2c_ws_w}\Gamma^{\phi^+W^-}_\nu(k+k_+,k_-) = 0
\end{aligned} \tag{A.25}
$$

$$
\begin{aligned}
&k_+^\mu \Gamma^{W^+A\phi^-}_{\mu\nu}(k_+,k,k_-) - M_W\Gamma^{\phi^+A\phi^-}_\nu(k_+,k,k_-) \\
&-e\Gamma^{W^+\phi^-}_\nu(k+k_+,k_-) + \frac{e}{2s_w}(\Gamma^{HA}_\nu(k_++k_-,k) + i\Gamma^{\chi A}_\nu(k_++k_-,k)) = 0
\end{aligned} \tag{A.26}
$$

$$
\begin{aligned}
&k_+^\mu \Gamma^{W^+Z\phi^-}_{\mu\nu}(k_+,k,k_-) - M_W\Gamma^{\phi^+Z\phi^-}_\nu(k_+,k,k_-) \\
&+e\frac{c_w}{s_w}\Gamma^{W^+\phi^-}_\nu(k+k_+,k_-) + \frac{e}{2s_w}(\Gamma^{HZ}_\nu(k_++k_-,k) + i\Gamma^{\chi Z}_\nu(k_++k_-,k)) = 0
\end{aligned} \tag{A.27}
$$

$$\begin{aligned} & k_+^\mu \Gamma^{W^+W^-H}_{\mu\nu}(k_+,k_-,k) - M_W \Gamma^{\phi^+W^-H}_{\nu}(k_+,k_-,k) \\ -\frac{e}{2s_w}\Gamma^{\phi^+W^-}_{\nu}(k+k_+,k_-) & + e(\Gamma^{AH}_{\nu}(k_++k_-,k) - \frac{c_w}{s_w}\Gamma^{ZH}_{\nu}(k_++k_-,k)) = 0 \end{aligned} \tag{A.28}$$

$$\begin{aligned} & k_+^\mu \Gamma^{W^+W^-\chi}_{\mu\nu}(k_+,k_-,k) - M_W \Gamma^{\phi^+W^-\chi}_{\nu}(k_+,k_-,k) \\ -\frac{ie}{2s_w}\Gamma^{\phi^+W^-}_{\nu}(k+k_+,k_-) & + e(\Gamma^{A\chi}_{\nu}(k_++k_-,k) - \frac{c_w}{s_w}\Gamma^{Z\chi}_{\nu}(k_++k_-,k)) = 0 \end{aligned} \tag{A.29}$$

$$\begin{aligned} & k_-^\mu \Gamma^{W^-A\phi^+}_{\mu\nu}(k_-,k,k_+) + M_W \Gamma^{\phi^-A\phi^+}_{\nu}(k_-,k,k_+) \\ +e\Gamma^{W^-\phi^+}_{\nu}(k+k_-,k_+) & - \frac{e}{2s_w}(\Gamma^{HA}_{\nu}(k_++k_-,k) - i\Gamma^{\chi A}_{\nu}(k_++k_-,k)) = 0 \end{aligned} \tag{A.30}$$

$$\begin{aligned} & k_-^\mu \Gamma^{W^-Z\phi^+}_{\mu\nu}(k_-,k,k_+) + M_W \Gamma^{\phi^-Z\phi^+}_{\nu}(k_-,k,k_+) \\ -e\frac{c_w}{s_w}\Gamma^{W^-\phi^+}_{\nu}(k+k_-,k_+) & - \frac{e}{2s_w}(\Gamma^{HZ}_{\nu}(k_++k_-,k) - i\Gamma^{\chi Z}_{\nu}(k_++k_-,k)) = 0 \end{aligned} \tag{A.31}$$

$$\begin{aligned} & k_-^\mu \Gamma^{W^-W^+H}_{\mu\nu}(k_-,k_+,k) + M_W \Gamma^{\phi^-W^+H}_{\nu}(k_-,k_+,k) \\ +\frac{e}{2s_w}\Gamma^{\phi^-W^+}_{\nu}(k+k_-,k_+) & - e(\Gamma^{AH}_{\nu}(k_++k_-,k) - \frac{c_w}{s_w}\Gamma^{ZH}_{\nu}(k_++k_-,k)) = 0 \end{aligned} \tag{A.32}$$

$$\begin{aligned} & k_-^\mu \Gamma^{W^-W^+\chi}_{\mu\nu}(k_-,k_+,k) + M_W \Gamma^{\phi^-W^+\chi}_{\nu}(k_-,k_+,k) \\ -\frac{ie}{2s_w}\Gamma^{\phi^-W^+}_{\nu}(k+k_-,k_+) & - e(\Gamma^{A\chi}_{\nu}(k_++k_-,k) - \frac{c_w}{s_w}\Gamma^{Z\chi}_{\nu}(k_++k_-,k)) = 0 \end{aligned} \tag{A.33}$$

A.5 Ward identities involving VSS, SSS and SS

$$k_1^\mu \Gamma^{AHH}_{\mu}(k_1,k_2,k_3) = k_1^\mu \Gamma^{AH\chi}_{\mu}(k_1,k_2,k_3) = k_1^\mu \Gamma^{A\chi\chi}_{\mu}(k_1,k_2,k_3) = 0 \tag{A.34}$$

$$k^\mu \Gamma^{A\phi^+\phi^-}_{\mu}(k,k_+,k_-) + e(\Gamma^{\phi^+\phi^-}(k+k_+,k_-) - \Gamma^{\phi^-\phi^+}(k+k_-,k_+)) = 0 \tag{A.35}$$

$$\begin{aligned} k_1^\mu \Gamma^{ZHH}_{\mu}(k_1,k_2,k_3) - iM_Z\Gamma^{\chi HH}(k_1,k_2,k_3) \; - \; & \frac{ie}{2c_w s_w}\Big(\Gamma^{\chi H}(k_1+k_2,k_3) \\ & + \; \Gamma^{\chi H}(k_1+k_3,k_2)\Big) = 0 \end{aligned} \tag{A.36}$$

$$\begin{aligned} k_1^\mu \Gamma^{ZH\chi}_{\mu}(k_1,k_2,k_3) - iM_Z\Gamma^{\chi H\chi}(k_1,k_2,k_3) \; - \; & \frac{ie}{2c_w s_w}\Big(\Gamma^{\chi\chi}(k_1+k_2,k_3) \\ & - \; \Gamma^{HH}(k_1+k_3,k_2)\Big) = 0 \end{aligned} \tag{A.37}$$

$$k_1^\mu \Gamma_\mu^{Z\chi\chi}(k_1,k_2,k_3) - iM_Z \Gamma^{\chi\chi\chi}(k_1,k_2,k_3) + \frac{ie}{2c_w s_w}\big(\Gamma^{H\chi}(k_1+k_2,k_3) + \Gamma^{H\chi}(k_1+k_3,k_2)\big) = 0 \tag{A.38}$$

$$k^\mu \Gamma_\mu^{Z\phi^+\phi^-}(k,k_+,k_-) - iM_Z \Gamma^{\chi\phi^+\phi^-}(k,k_+,k_-) - e\frac{c_w^2 - s_w^2}{2c_w s_w}(\Gamma^{\phi^+\phi^-}(k+k_+,k_-) - \Gamma^{\phi^-\phi^+}(k+k_-,k_+)) = 0 \tag{A.39}$$

$$k_+^\mu \Gamma_\mu^{W^+H\phi^-}(k_+,k,k_-) - M_W \Gamma^{\phi^+H\phi^-}(k_+,k,k_-) + \frac{e}{2s_w}(\Gamma^{HH}(k_-+k_+,k) + i\Gamma^{\chi H}(k_++k_-,k)) - \frac{e}{2s_w}\Gamma^{\phi^+\phi^-}(k_++k,k_-) = 0 \tag{A.40}$$

$$k_+^\mu \Gamma_\mu^{W^+\chi\phi^-}(k_+,k,k_-) - M_W \Gamma^{\phi^+\chi\phi^-}(k_+,k,k_-) + \frac{e}{2s_w}(\Gamma^{H\chi}(k_-+k_+,k) + i\Gamma^{\chi\chi}(k_++k_-,k)) - \frac{ie}{2s_w}\Gamma^{\phi^+\phi^-}(k_++k,k_-) = 0 \tag{A.41}$$

$$k_-^\mu \Gamma_\mu^{W^-H\phi^+}(k_-,k,k_+) + M_W \Gamma^{\phi^-H\phi^+}(k_-,k,k_+) - \frac{e}{2s_w}(\Gamma^{HH}(k_-+k_+,k) - i\Gamma^{\chi H}(k_++k_-,k)) + \frac{e}{2s_w}\Gamma^{\phi^-\phi^+}(k_-+k,k_+) = 0 \tag{A.42}$$

$$k_-^\mu \Gamma_\mu^{W^-\chi\phi^+}(k_-,k,k_+) + M_W \Gamma^{\phi^-\chi\phi^+}(k_-,k,k_+) - \frac{e}{2s_w}(\Gamma^{H\chi}(k_-+k_+,k) - i\Gamma^{\chi\chi}(k_++k_-,k)) - \frac{ie}{2s_w}\Gamma^{\phi^-\phi^+}(k_-+k,k_+) = 0 \tag{A.43}$$

A.6 Ward identities involving VVVV, VVVS and VVV

$$k_{1,2,3,4}^\mu \Gamma_{\mu\nu\kappa\sigma}^{AAAA}(k_1,k_2,k_3,k_4) = 0 \tag{A.44}$$

$$k_{1,2,3}^\mu \Gamma_{\mu\nu\kappa\sigma}^{AAAZ}(k_1,k_2,k_3,k_4) = 0 \tag{A.45}$$

$$k_{1,2}^\mu \Gamma_{\mu\nu\kappa\sigma}^{AAZZ}(k_1,k_2,k_3,k_4) = 0 \tag{A.46}$$

$$k_1^\mu \Gamma_{\mu\nu\kappa\sigma}^{AZZZ}(k_1,k_2,k_3,k_4) = 0 \tag{A.47}$$

$$k_1^\mu \Gamma^{AAW^+W^-}_{\mu\nu\kappa\sigma}(k_1,k_2,k_+,k_-) + \; e \; \Big[\Gamma^{AW^+W^-}_{\nu\kappa\sigma}(k_2,k_1+k_+,k_-) - \; \Gamma^{AW^+W^-}_{\nu\kappa\sigma}(k_2,k_+,k_1+k_-)\Big] = 0 \tag{A.48}$$

$$k_1^\mu \Gamma^{AZW^+W^-}_{\mu\nu\kappa\sigma}(k_1,k_2,k_+,k_-) + \; e \; \Big[\Gamma^{W^+ZW^-}_{\kappa\nu\sigma}(k_1+k_+,k_2,k_-) - \; \Gamma^{W^-ZW^+}_{\sigma\nu\kappa}(k_1+k_-,k_2,k_+)\Big] = 0 \tag{A.49}$$

$$k^\mu \Gamma^{ZV_2V_3V_4}_{\mu\nu\kappa\sigma}(k_1,k_2,k_3,k_4) - iM_Z \Gamma^{\chi V_2V_3V_4}_{\nu\kappa\sigma}(k_1,k_2,k_3,k_4) = 0\,, \tag{A.50}$$

where k here refers to any of the Z momenta and V's stand for A or Z.

$$\begin{aligned} k_+^\mu \; \Gamma^{W^+W^-AA}_{\mu\nu\kappa\sigma}(k_+,k_-,k_3,k_4) &+ e\Big[\Gamma^{AAA}_{\nu\kappa\sigma}(k_+ + k_-,k_3,k_4) - \frac{c_w}{s_w}\Gamma^{ZAA}_{\nu\kappa\sigma}(k_+ + k_-,k_3,k_4) \\ - \; \Gamma^{W^+W^-A}_{\sigma\nu\kappa}(k_+ + k_4,k_-,k_3) &- \Gamma^{W^-W^+A}_{\nu\kappa\sigma}(k_-,k_+ + k_3,k_4)\Big] \\ - \; M_W \Gamma^{\phi^+W^-AA}_{\nu\kappa\sigma}(k_+,k_-,k_3,k_4) &= 0 \end{aligned} \tag{A.51}$$

$$\begin{aligned} k_-^\mu \; \Gamma^{W^-W^+AA}_{\mu\nu\kappa\sigma}(k_-,k_+,k_3,k_4) &- e\Big[\Gamma^{AAA}_{\nu\kappa\sigma}(k_+ + k_-,k_3,k_4) - \frac{c_w}{s_w}\Gamma^{ZAA}_{\nu\kappa\sigma}(k_+ + k_-,k_3,k_4) \\ - \; \Gamma^{W^-W^+A}_{\sigma\nu\kappa}(k_- + k_4,k_+,k_3) &- \Gamma^{W^+W^-A}_{\nu\kappa\sigma}(k_+,k_- + k_3,k_4)\Big] \\ + \; M_W \Gamma^{\phi^-W^+AA}_{\nu\kappa\sigma}(k_-,k_+,k_3,k_4) &= 0 \end{aligned} \tag{A.52}$$

$$\begin{aligned} k_1^\mu \; \Gamma^{ZZW^+W^-}_{\mu\nu\kappa\sigma}(k_1,k_2,k_+,k_-) &- e\frac{c_w}{s_w}\Big[\Gamma^{W^+ZW^-}_{\kappa\nu\sigma}(k_1+k_+,k_2,k_-) \\ - \; \Gamma^{W^-ZW^+}_{\sigma\nu\kappa}(k_1+k_-,k_2,k_+)\Big] &- iM_Z\Gamma^{\chi ZW^+W^-}_{\nu\kappa\sigma}(k_1,k_2,k_+,k_-) = 0 \end{aligned} \tag{A.53}$$

$$\begin{aligned} k_+^\mu \; \Gamma^{W^+ZZW^-}_{\mu\nu\kappa\sigma}(k_+,k_1,k_2,k_-) &+ e\Big[\Gamma^{AZZ}_{\sigma\nu\kappa}(k_+ + k_-,k_1,k_2) - \frac{c_w}{s_w}\Gamma^{ZZZ}_{\sigma\nu\kappa}(k_+ + k_-,k_1,k_2) \\ + \; \frac{c_w}{s_w}\Gamma^{W^+ZW^-}_{\nu\kappa\sigma}(k_+ + k_1,k_2,k_-) &+ \frac{c_w}{s_w}\Gamma^{W^+ZW^-}_{\kappa\nu\sigma}(k_+ + k_2,k_1,k_-)\Big] \\ - \; M_W\Gamma^{\phi^+ZZW^-}_{\nu\kappa\sigma}(k_+,k_1,k_2,k_-) &= 0 \end{aligned} \tag{A.54}$$

$$\begin{aligned} k_-^\mu \; \Gamma^{W^-W^+ZZ}_{\mu\nu\kappa\sigma}(k_-,k_+,k_3,k_4) &- e\Big[\Gamma^{AZZ}_{\nu\kappa\sigma}(k_+ + k_-,k_3,k_4) - \frac{c_w}{s_w}\Gamma^{ZZZ}_{\nu\kappa\sigma}(k_+ + k_-,k_3,k_4) \\ + \; \frac{c_w}{s_w}\Gamma^{W^-W^+Z}_{\kappa\nu\sigma}(k_- + k_3,k_+,k_4) &+ \frac{c_w}{s_w}\Gamma^{W^-W^+Z}_{\sigma\nu\kappa}(k_- + k_4,k_+,k_3)\Big] \\ + \; M_W\Gamma^{\phi^-W^+ZZ}_{\nu\kappa\sigma}(k_-,k_+,k_3,k_4) &= 0 \end{aligned} \tag{A.55}$$

$$k_1^\mu\ \Gamma^{ZAW^+W^-}_{\mu\nu\kappa\sigma}(k_1,k_2,k_+,k_-) - e\frac{c_w}{s_w}\Big[\Gamma^{W^+AW^-}_{\kappa\nu\sigma}(k_1+k_+,k_2,k_-)$$
$$-\ \Gamma^{W^-AW^+}_{\sigma\nu\kappa}(k_1+k_-,k_2,k_+)\Big] - iM_Z\Gamma^{\chi AW^+W^-}_{\nu\kappa\sigma}(k_1,k_2,k_+,k_-) = 0 \tag{A.56}$$

$$k_+^\mu\ \Gamma^{W^+W^-AZ}_{\mu\nu\kappa\sigma}(k_+,k_-,k_3,k_4) + e\Bigg[\Gamma^{AAZ}_{\nu\kappa\sigma}(k_++k_-,k_3,k_4) - \frac{c_w}{s_w}\Gamma^{ZAZ}_{\nu\kappa\sigma}(k_++k_-,k_3,k_4)$$
$$-\ \Gamma^{W^+W^-Z}_{\kappa\nu\sigma}(k_++k_3,k_-,k_4) + \frac{c_w}{s_w}\Gamma^{W^+W^-A}_{\sigma\nu\kappa}(k_++k_4,k_-,k_3)\Bigg]$$
$$-\ M_W\Gamma^{\phi^+W^-AZ}_{\nu\kappa\sigma}(k_+,k_-,k_3,k_4) = 0 \tag{A.57}$$

$$k_-^\mu\ \Gamma^{W^-W^+AZ}_{\mu\nu\kappa\sigma}(k_-,k_+,k_3,k_4) - e\Bigg[\Gamma^{AAZ}_{\nu\kappa\sigma}(k_++k_-,k_3,k_4) - \frac{c_w}{s_w}\Gamma^{ZAZ}_{\nu\kappa\sigma}(k_++k_-,k_3,k_4)$$
$$-\ \Gamma^{W^-W^+Z}_{\kappa\nu\sigma}(k_-+k_3,k_+,k_4) + \frac{c_w}{s_w}\Gamma^{W^-W^+A}_{\sigma\nu\kappa}(k_-+k_4,k_+,k_3)\Bigg]$$
$$+\ M_W\Gamma^{\phi^-W^+AZ}_{\nu\kappa\sigma}(k_-,k_+,k_3,k_4) = 0 \tag{A.58}$$

$$k_{1+}^\mu\ \Gamma^{W^+W^-W^+W^-}_{\mu\nu\kappa\sigma}(k_{1+},k_{1-},k_{2+},k_{2-}) + e\Big[\Gamma^{AW^+W^-}_{\nu\kappa\sigma}(k_{1+}+k_{1-},k_{2+},k_{2-})$$
$$-\ \frac{c_w}{s_w}\Gamma^{ZW^+W^-}_{\nu\kappa\sigma}(k_{1+}+k_{1-},k_{2+},k_{2-}) + \Gamma^{AW^-W^+}_{\sigma\nu\kappa}(k_{1+}+k_{2-},k_{1-},k_{2+})$$
$$-\ \frac{c_w}{s_w}\Gamma^{ZW^-W^+}_{\sigma\nu\kappa}(k_{1+}+k_{2-},k_{1-},k_{2+})\Big] - M_W\Gamma^{\phi^+W^-W^+W^-}_{\nu\kappa\sigma}(k_{1+},k_{1-},k_{2+},k_{2-}) = 0 \tag{A.59}$$

$$k_{1-}^\mu\ \Gamma^{W^-W^+W^-W^+}_{\mu\nu\kappa\sigma}(k_{1-},k_{1+},k_{2-},k_{2+}) - e\Big[\Gamma^{AW^-W^+}_{\nu\kappa\sigma}(k_{1+}+k_{1-},k_{2-},k_{2+})$$
$$-\ \frac{c_w}{s_w}\Gamma^{ZW^-W^+}_{\nu\kappa\sigma}(k_{1+}+k_{1-},k_{2-},k_{2+}) + \Gamma^{AW^+W^-}_{\sigma\nu\kappa}(k_{2+}+k_{1-},k_{1+},k_{2-})$$
$$-\ \frac{c_w}{s_w}\Gamma^{ZW^+W^-}_{\sigma\nu\kappa}(k_{2+}+k_{1-},k_{1+},k_{2-})\Big] + M_W\Gamma^{\phi^-W^+W^-W^+}_{\nu\kappa\sigma}(k_{1-},k_{1+},k_{2-},k_{2+}) = 0 \tag{A.60}$$

A.7 Ward identities involving SSSS, VSSS and SSS

$$k_1^\mu\ \Gamma^{Z\chi HH}_{\mu}(k_1,k_2,k_3,k_4) - iM_Z\Gamma^{\chi\chi HH}(k_1,k_2,k_3,k_4) - \frac{ie}{2c_ws_w}\big[\Gamma^{\chi\chi H}(k_1+k_3,k_2,k_4)$$
$$+\ \Gamma^{\chi\chi H}(k_1+k_4,k_2,k_3) - \Gamma^{HHH}(k_1+k_2,k_3,k_4)\big] = 0 \tag{A.61}$$

$$k_1^\mu\ \Gamma_\mu^{Z\chi\chi\chi}(k_1,k_2,k_3,k_4) - iM_Z\Gamma^{\chi\chi\chi\chi}(k_1,k_2,k_3,k_4) + \frac{ie}{2c_ws_w}\Big[\Gamma^{H\chi\chi}(k_1+k_2,k_3,k_4)$$
$$+\ \Gamma^{H\chi\chi}(k_1+k_3,k_2,k_4) + \Gamma^{H\chi\chi}(k_1+k_4,k_2,k_3)\Big] = 0 \tag{A.62}$$

$$\begin{aligned}
&k_1^\mu\ \Gamma_\mu^{ZH\phi^+\phi^-}(k_1,k_2,k_+,k_-) - iM_Z\Gamma^{\chi H\phi^+\phi^-}(k_1,k_2,k_+,k_-)\\
&-\ e\frac{c_w^2-s_w^2}{2c_ws_w}\Big[\Gamma^{\phi^+H\phi^-}(k_1+k_+,k_2,k_-) - \Gamma^{\phi^-H\phi^+}(k_1+k_-,k_2,k_+)\Big]\\
&-\ \frac{ie}{2c_ws_w}\Gamma^{\chi\phi^+\phi^-}(k_1+k_2,k_+,k_-) = 0
\end{aligned} \tag{A.63}$$

$$\begin{aligned}
&k_1^\mu\ \Gamma_\mu^{Z\chi\phi^+\phi^-}(k_1,k_2,k_+,k_-) - iM_Z\Gamma^{\chi\chi\phi^+\phi^-}(k_1,k_2,k_+,k_-)\\
&-\ e\frac{c_w^2-s_w^2}{2c_ws_w}\Big[\Gamma^{\phi^+\chi\phi^-}(k_1+k_+,k_2,k_-) - \Gamma^{\phi^-\chi\phi^+}(k_1+k_-,k_2,k_+)\Big]\\
&+\ \frac{ie}{2c_ws_w}\Gamma^{H\phi^+\phi^-}(k_1+k_2,k_+,k_-) = 0
\end{aligned} \tag{A.64}$$

$$\begin{aligned}
&k_+^\mu\ \Gamma_\mu^{W^+\phi^-HH}(k_+,k_-,k_1,k_2) - M_W\Gamma^{\phi^+\phi^-HH}(k_+,k_-,k_1,k_2)\\
&+\ \frac{e}{2s_w}\Big[\Gamma^{HHH}(k_++k_-,k_1,k_2) + i\Gamma^{\chi HH}(k_++k_-,k_1,k_2)\\
&-\ \Gamma^{\phi^+\phi^-H}(k_1+k_+,k_-,k_2) - \Gamma^{\phi^+\phi^-H}(k_2+k_+,k_-,k_1)\Big] = 0
\end{aligned} \tag{A.65}$$

$$k_+^\mu\ \Gamma_\mu^{W^+\phi^-H\chi}(k_+,k_-,k_1,k_2) - M_W\Gamma^{\phi^+\phi^-H\chi}(k_+,k_-,k_1,k_2) + \frac{e}{2s_w}\Big[\Gamma^{HH\chi}(k_++k_-,k_1,k_2)$$
$$+\ i\Gamma^{\chi H\chi}(k_++k_-,k_1,k_2) - \Gamma^{\phi^+\phi^-\chi}(k_1+k_+,k_-,k_2) - i\Gamma^{\phi^+\phi^-H}(k_2+k_+,k_-,k_1)\Big] = 0 \tag{A.66}$$

$$\begin{aligned}
&k_+^\mu\ \Gamma_\mu^{W^+\phi^-\chi\chi}(k_+,k_-,k_1,k_2) - M_W\Gamma^{\phi^+\phi^-\chi\chi}(k_+,k_-,k_1,k_2)\\
&+\ \frac{e}{2s_w}\Big[\Gamma^{H\chi\chi}(k_++k_-,k_1,k_2) + i\Gamma^{\chi\chi\chi}(k_++k_-,k_1,k_2)\\
&-\ i\,\Gamma^{\phi^+\phi^-\chi}(k_1+k_+,k_-,k_2) - i\Gamma^{\phi^+\phi^-\chi}(k_2+k_+,k_-,k_1)\Big] = 0
\end{aligned} \tag{A.67}$$

$$\begin{aligned}
&k_{1+}^\mu\ \Gamma_\mu^{W^+\phi^-\phi^+\phi^-}(k_{1+},k_{1-},k_{2+},k_{2-}) - M_W\Gamma^{\phi^+\phi^-\phi^+\phi^-}(k_{1+},k_{1-},k_{2+},k_{2-})\\
&+\ \frac{e}{2s_w}\Big[\Gamma^{H\phi^+\phi^-}(k_{1+}+k_{1-},k_{2+},k_{2-}) + i\Gamma^{\chi\phi^+\phi^-}(k_{1+}+k_{1-},k_{2+},k_{2-})\\
&+\ \Gamma^{H\phi^-\phi^+}(k_{1+}+k_{2-},k_{1-},k_{2+}) + i\Gamma^{\chi\phi^-\phi^+}(k_{1+}+k_{2-},k_{1-},k_{2+})\Big] = 0
\end{aligned} \tag{A.68}$$

$$
\begin{aligned}
k_-^{\mu}\ & \Gamma_{\mu}^{W^-\phi^+HH}(k_-,k_+,k_1,k_2)+M_W\Gamma^{\phi^-\phi^+HH}(k_-,k_+,k_1,k_2)\\
-\ & \frac{e}{2s_w}\Big[\Gamma^{HHH}(k_++k_-,k_1,k_2)-i\Gamma^{\chi HH}(k_++k_-,k_1,k_2)\\
-\ & \Gamma^{\phi^-\phi^+H}(k_1+k_-,k_+,k_2)-\Gamma^{\phi^-\phi^+H}(k_2+k_-,k_+,k_1)\Big]=0
\end{aligned}
\tag{A.69}
$$

$$
\begin{aligned}
k_-^{\mu}\ & \Gamma_{\mu}^{W^-\phi^+H\chi}(k_-,k_+,k_1,k_2)+M_W\Gamma^{\phi^-\phi^+H\chi}(k_-,k_+,k_1,k_2)\\
-\ & \frac{e}{2s_w}\Big[\Gamma^{HH\chi}(k_++k_-,k_1,k_2)-i\Gamma^{\chi H\chi}(k_++k_-,k_1,k_2)\\
-\ & \Gamma^{\phi^-\phi^+\chi}(k_1+k_-,k_+,k_2)+i\Gamma^{\phi^-\phi^+H}(k_2+k_-,k_+,k_1)\Big]=0
\end{aligned}
\tag{A.70}
$$

$$
\begin{aligned}
k_-^{\mu}\ & \Gamma_{\mu}^{W^-\phi^+\chi\chi}(k_-,k_+,k_1,k_2)+M_W\Gamma^{\phi^-\phi^+\chi\chi}(k_-,k_+,k_1,k_2)\\
-\ & \frac{e}{2s_w}\Big[\Gamma^{H\chi\chi}(k_++k_-,k_1,k_2)-i\Gamma^{\chi\chi\chi}(k_++k_-,k_1,k_2)\\
+\ & i\,\Gamma^{\phi^-\phi^+\chi}(k_1+k_-,k_+,k_2)+i\Gamma^{\phi^-\phi^+\chi}(k_2+k_-,k_+,k_1)\Big]=0
\end{aligned}
\tag{A.71}
$$

$$
\begin{aligned}
k_{1-}^{\mu}\ & \Gamma_{\mu}^{W^-\phi^+\phi^-\phi^+}(k_{1-},k_{1+},k_{2-},k_{2+})+M_W\Gamma^{\phi^-\phi^+\phi^-\phi^+}(k_{1-},k_{1+},k_{2-},k_{2+})\\
-\ & \frac{e}{2s_w}\Big[\Gamma^{H\phi^-\phi^+}(k_{1+}+k_{1-},k_{2-},k_{2+})-i\Gamma^{\chi\phi^-\phi^+}(k_{1+}+k_{1-},k_{2-},k_{2+})\\
+\ & \Gamma^{H\phi^+\phi^-}(k_{2+}+k_{1-},k_{1+},k_{2-})-i\Gamma^{\chi\phi^+\phi^-}(k_{2+}+k_{1-},k_{1+},k_{2-})\Big]=0
\end{aligned}
\tag{A.72}
$$

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